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United States Department of Agriculture
Soil Conservation Service
Engineering Division

TENTATIVE
Technical Release No. 2
Design Section
October 1, 1956

EARTH SPILLWAYS[1]

1. Definition. An earth spillway is an open channel, usually trapezoidal in cross section. It consists of an inlet channel, control section, and exit channel. It should be lined with erosion-resisting vegetation, but the absence of vegetative lining does not preclude the use of emergency earth spillways where their use to carry discharge past the dam is very infrequent.

2. Uses. Such channels are commonly used as emergency spillways. Emergency spillways are designed for infrequent use. They assist the mechanical principal spillway in conveying large flood flows safely past an earth dam embankment.

Earth spillways, lined with erosion-resisting vegetation, are sometimes used on small dams in small watersheds as the principal spillway; i.e., a spillway which comes into operation frequently and conveys the discharge at non-erosive velocities to a junction with the original waterway below the dam.

The use of an earth spillway as a principal spillway should be supplemented by a mechanical spillway that will convey small amounts of runoff and keep the vegetated earth spillway free of prolonged flows which tend to prevent vigorous growth of the vegetation in it. The difference in elevation between the crest of such a spillway and the crest of the earth spillway should be sufficient to make the mechanical spillway operate efficiently before flow takes place in the earth spillway.

3. Layout. Earth spillways are usually located in undisturbed earth in the abutments at one or both ends of an earth embankment or over a topographic saddle at some point on the periphery of the reservoir. The channel should be excavated into undisturbed earth or rock and the water surface under maximum possible discharge should be confined by undisturbed earth or rock or carefully constructed and compacted levees of safe proportions. In no case should the water surface be permitted against loose uncompacted fill.

The inlet channel is defined as that part of the spillway upstream from the control section where the flow passes through critical depth. The exit channel is that part of the spillway downstream from the control section. In the inlet channel the flow is subcritical and in the exit channel the flow is supercritical. The slope below the control section must be such that the control section remains fixed in position for all discharges significant in the hydraulic design of the spillway. Curvature is permissible in those reaches of the spillway where subcritical flow exists for the full range of discharge;

[1]This Technical Release was developed and written by M. M. Culp, Head, Design Section, with assistance from A. R. Gregory.

where supercritical flow exists, the channel must be straight in alignment to a point far enough below the earth dam embankment to insure that any flow which might escape from the exit channel cannot do damage to the earth dam.

Ignoring construction problems and cost, the control section may be placed at any point along the spillway. Generally speaking, the farther downstream the control section is located, the higher the required stage in the reservoir for any given discharge because of the increase in head loss through the increased length of the approach channel which results in a corresponding increase in the elevation of top of embankment and higher cost. The cross-sectional area of flow in the inlet channel should be large in comparison to the flow area at the control section so as to minimize losses through the inlet channel and reduce the required height of the earth dam embankment. The desired increase in flow area through the inlet channel may be obtained by increasing its depth or width or by a combination of the two.

Seldom will it be necessary or advisable to locate the control section as far as 100 feet below the intersection of the center line of the earth spillway and the center line of the earth dam embankment; hence, it is suggested that most of the earth spillway between the intersection mentioned above and the control section be constructed on a level or an adverse (negative) slope. Starting 20 to 30 feet above the control section, the depth of the channel may be increased significantly to increase flow area and reduce friction loss. This may be accomplished by inserting a short reach on an adverse 5 to 1 slope. The remaining upstream length of the approach channel may be placed on a level or an adverse slope. Where the depth of the channel varies as through the 5 to 1 slope, the bottom width of the channel should be changed to avoid abrupt changes in the configuration of the sloping channel banks. For example, if the width of the channel bottom just downstream from the 5 to 1 slope were 50 feet, the channel side slopes were 3 to 1 and the drop through the 5 to 1 bottom slope were 4 feet, the bottom width of the channel just upstream from the 5 to 1 bottom slope should be $50 - 2\ (3 \times 4) = 26$ feet. A uniform transition should exist between these two bottom widths.

As pointed out previously, the exit channel below the control section must be straight for a minimum distance depending upon the site conditions to avoid non-uniform flow distribution across the width of the channel and possible overtopping of retaining levees on the earth fill side of the channel. The cross section and slope of the exit channel must be such that the maximum possible discharge through the earth spillway is confined in the channel far enough that discharge from the earth spillway will not reach the gutter between the earth embankment and its abutment or any part of the earth embankment except as backwater in the channel below the dam.

Roads should not be graded up along or across earth spillways. If a roadway must cross the earth spillway, it should be located at least twenty feet upstream from the control section and preferably across a deepened part of the inlet channel.

Fences must be kept out of the spillway; a wire netting or field fence makes an excellent debris catcher and must not be placed across the earth spillway. If it is absolutely necessary to cross the spillway with a fence, suspended water gates that will rise and float on the surface of the water should be provided and kept in good repair, or the fence should be located at a low elevation in the approach channel just above normal pool level, so that debris will float over it when the emergency spillway comes into action.

Earth spillways should be made as wide as economics and the topography of the site will permit. Generally speaking for a given maximum design flood an increase in width of the earth spillway will permit a reduction in the height of the earth dam embankment and provide a greater reserve capacity of the earth spillway. This greater reserve capacity results from encroachment of reservoir stage on the freeboard provided for waves or frost action. In many locations, particularly where the abutment slopes are not too steep and the material excavated from the earth spillway can be used in the earth dam embankment, it will be found that wide earth spillways are economical.

Typical layouts of earth spillways are given on drawing ES-99. These layouts have been made on the assumption that the excavated earth is suitable for use in the earth dam embankment; hence, they do not necessarily represent attempts to accomplish layouts with a minimum amount of excavation. Since the cost of the earth spillway is but a part of the cost of the total job, and because the design of the earth spillway is directly related to the design of other parts of the structure, minimum cost studies of the earth spillway alone are usually not significant.

Typical profiles along the center line of earth spillways are shown on drawing ES-100. Adverse slopes in the inlet channel should not be steeper than 5 horizontal to 1 vertical.

Erosion-resistant vegetation should be established on the bottom and side slopes of earth spillways. To assist in this establishment, it is sometimes desirable and practical to divert surface runoff away from the cut slope on the abutment side of the channel by terracing or a diversion ditch. A spillway of similar layout, excavated partly or wholly in rock, may be used where it is economically justified and will provide a practical solution to the spillway problem at a particular site.

4. Investigation of Site. Topography and information on soils essential to proper layout and design of an earth spillway should be obtained along with the surveys made for the design of the dam. Obviously a satisfactory layout and hydraulic design cannot be made for an earth spillway without accurate and complete topography of all practical sites. Too often the topography obtained at a site does not cover sufficient area to permit proper layout studies and the preparation of final grading plans and estimates of quantities of earth or rock excavation.

Topographic maps having a scale of 1 inch = 40, 50, or 60 feet and a vertical interval of 1, 2, or 4 feet have been found to be most satisfactory for earth spillway layouts. A 1- or 2-foot vertical interval should be used

where the abutment slopes are less than about 30 percent; where the abutment slopes on the area under consideration for the earth spillway exceed 30 percent, a 4-foot vertical interval may be desirable.

Borings, soil classification, and tests are necessary to define

(a) the suitability of the material to be excavated for use in the earth dam embankment.

(b) the suitability of the earth at and slightly below the grade line of the earth spillway for the establishment and maintenance of vegetative cover. It may be necessary to over-excavate and backfill to grade with top soil.

(c) the location of rock or other critical strata such as sand in the soil profile. Rock might exist in sufficient quantity to preclude the use of such a spillway, cause a change in site or justify an increase in spillway storage and in the capacity of the mechanical spillway sufficient to care for the maximum design storm.

(d) the location of the water table or water-bearing strata and to facilitate the design of any abutment slope drainage that might be needed.

(e) the resistance of the slope on the abutment side of the earth spillway against sliding or sloughing.

(f) methods of correction for potential slides.

For example should loose, sterile sand be found at or near the proposed grade line, it probably would be necessary to

(a) investigate other sites.

(b) design the mechanical spillway and storage requirements to care for all of the maximum design storms.

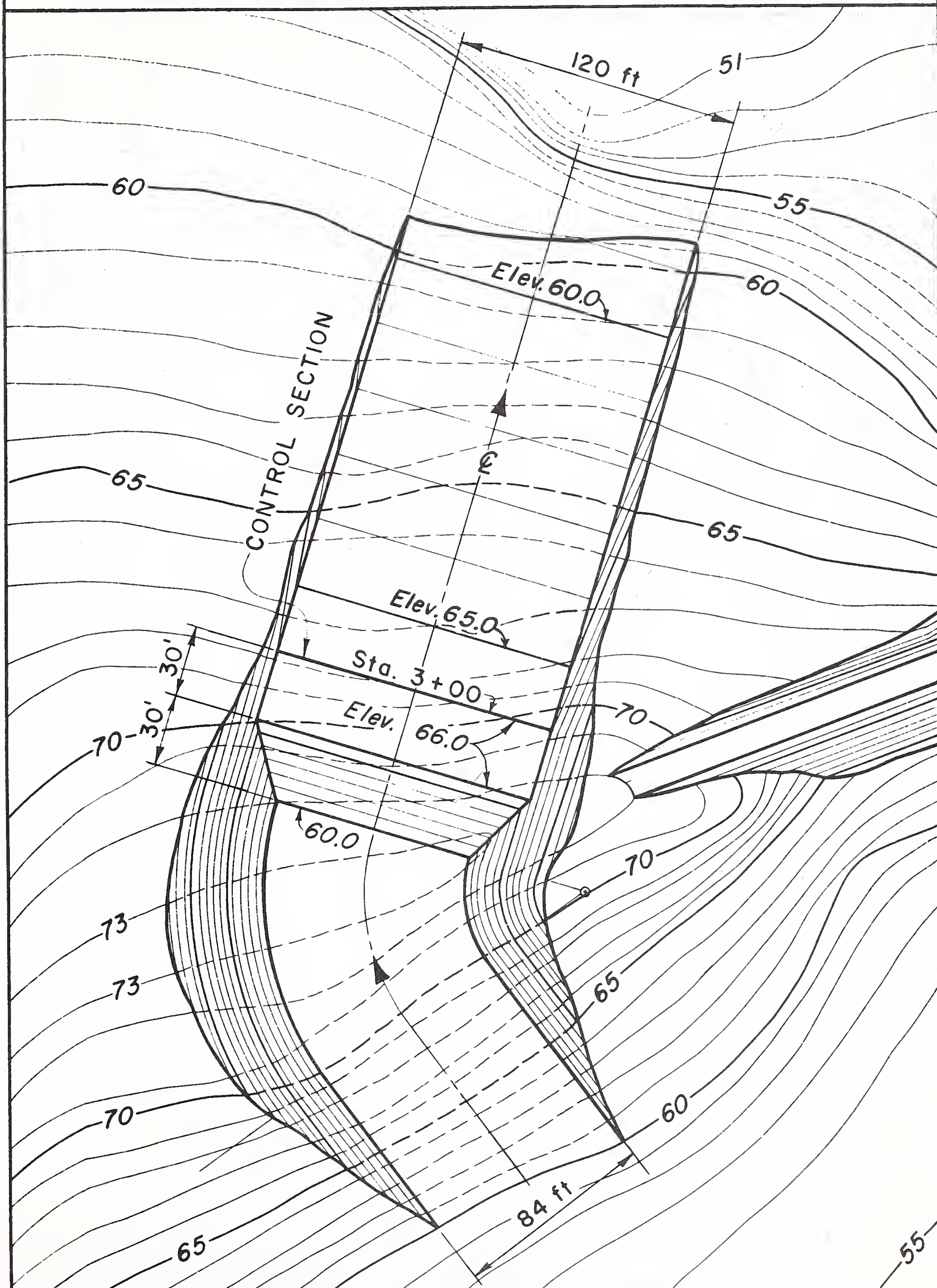
(c) drastically reduce the permissible velocities in the earth spillway.

5. Hydraulic Design. The methods recommended for use in the hydraulic design of earth spillways depend upon whether or not the effect of spillway storage is to be included in the design.

The only precise way to analyze the passage of flood flow through an earth spillway involves flood routing which requires a stage-discharge curve for the spillway. Any design which deviates from this basic procedure is an approximation of the truth. For small dams of low failure hazard, it is practical to adopt approximate design procedures.

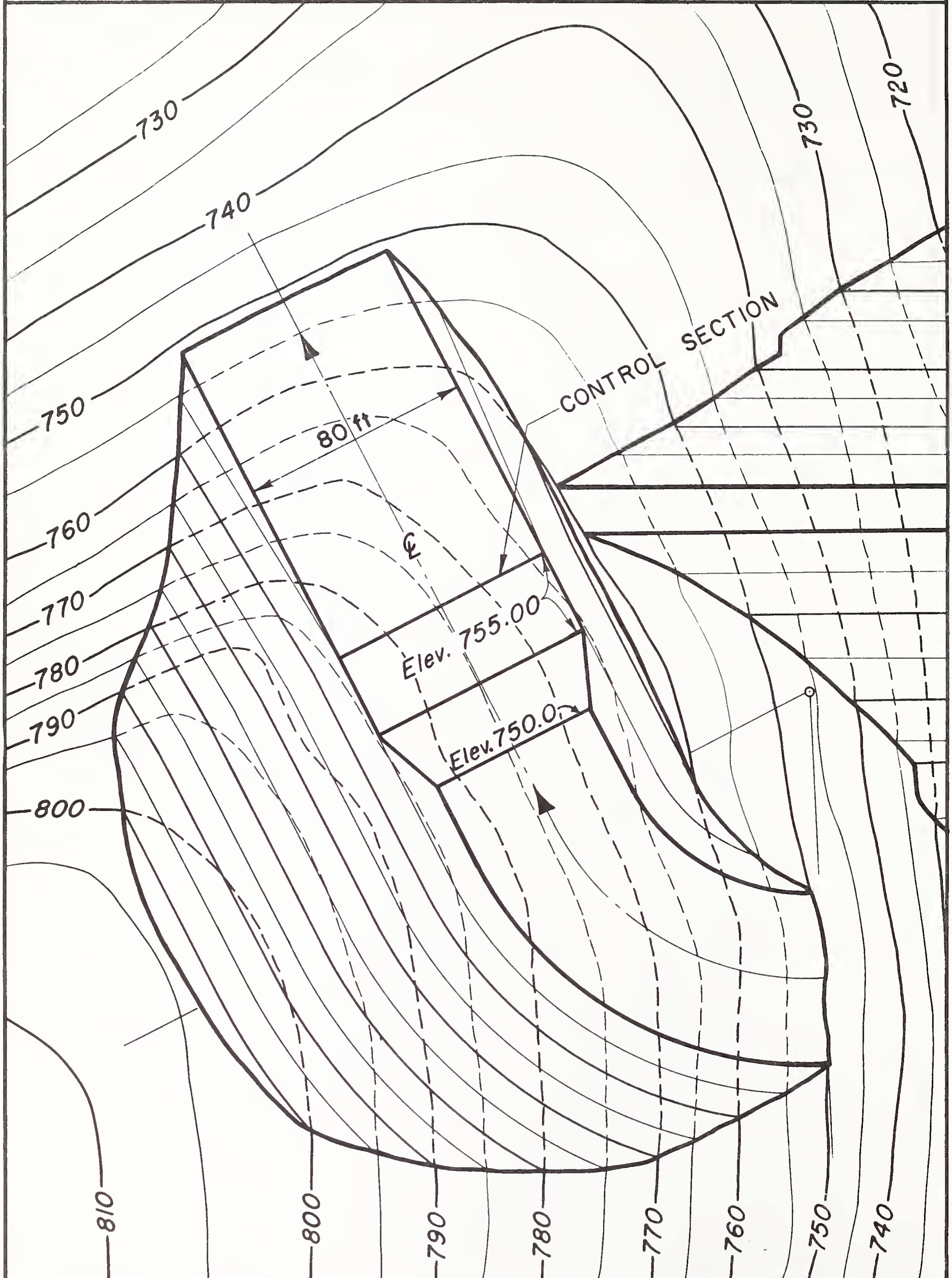
It is desirable to have available methods or equations which will give an approximate answer which can be used to guide a more refined design process where it is needed. The assumption of an equivalent rectangular control section and exit channel with frictionless sidewalls leads to equations that are helpful. Under such assumptions the depth of flow d and the hydraulic radius r are equal.

EARTH SPILLWAYS : Typical layout and grading plans.



REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION-DESIGN SECTION	STANDARD DWG. NO. ES- 99 SHEET 1 OF 5 DATE 4-12-55

EARTH SPILLWAYS : Typical layout and grading plans.



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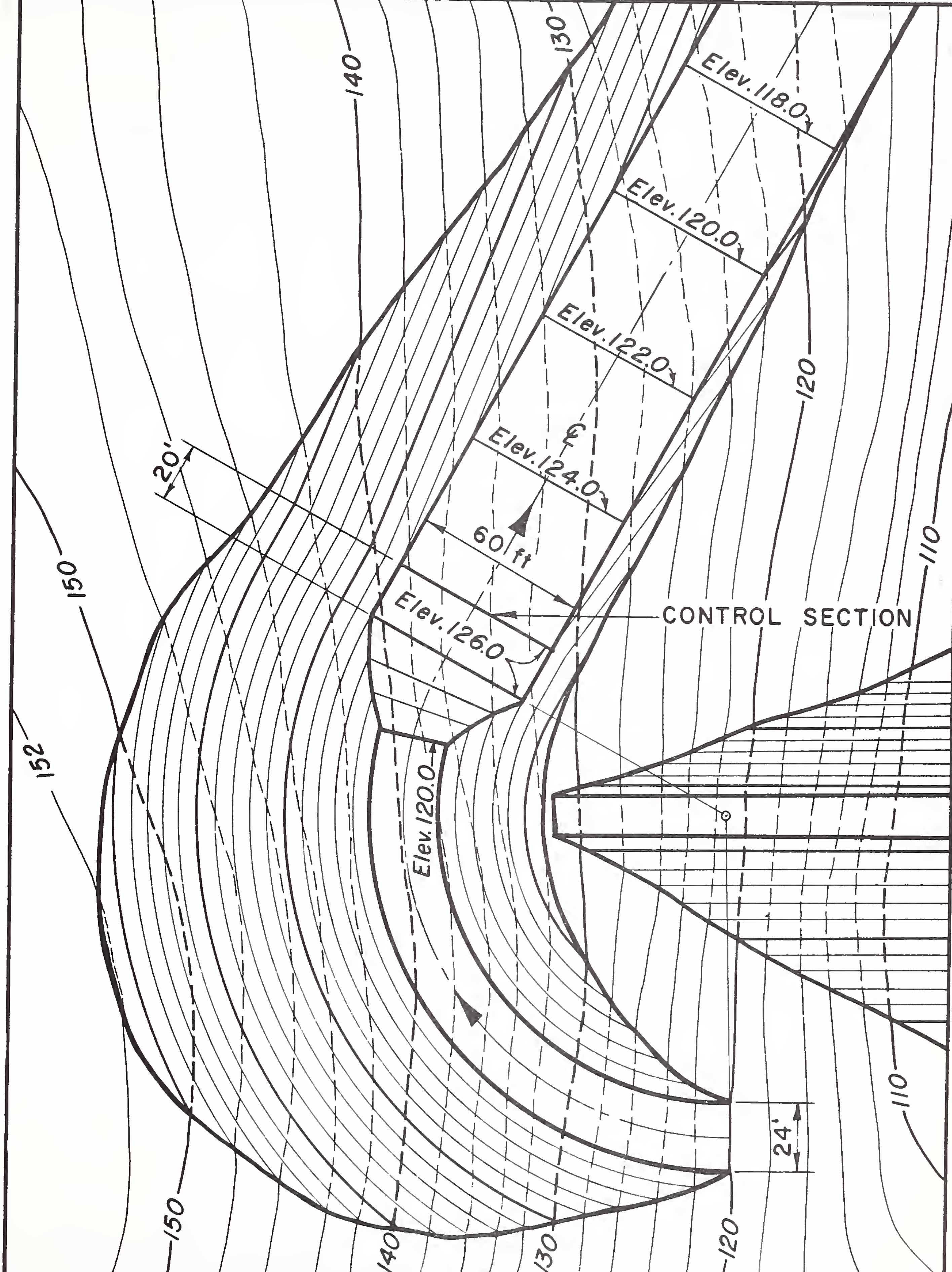
EARTH SPILLWAYS : Typical layout and grading plans.



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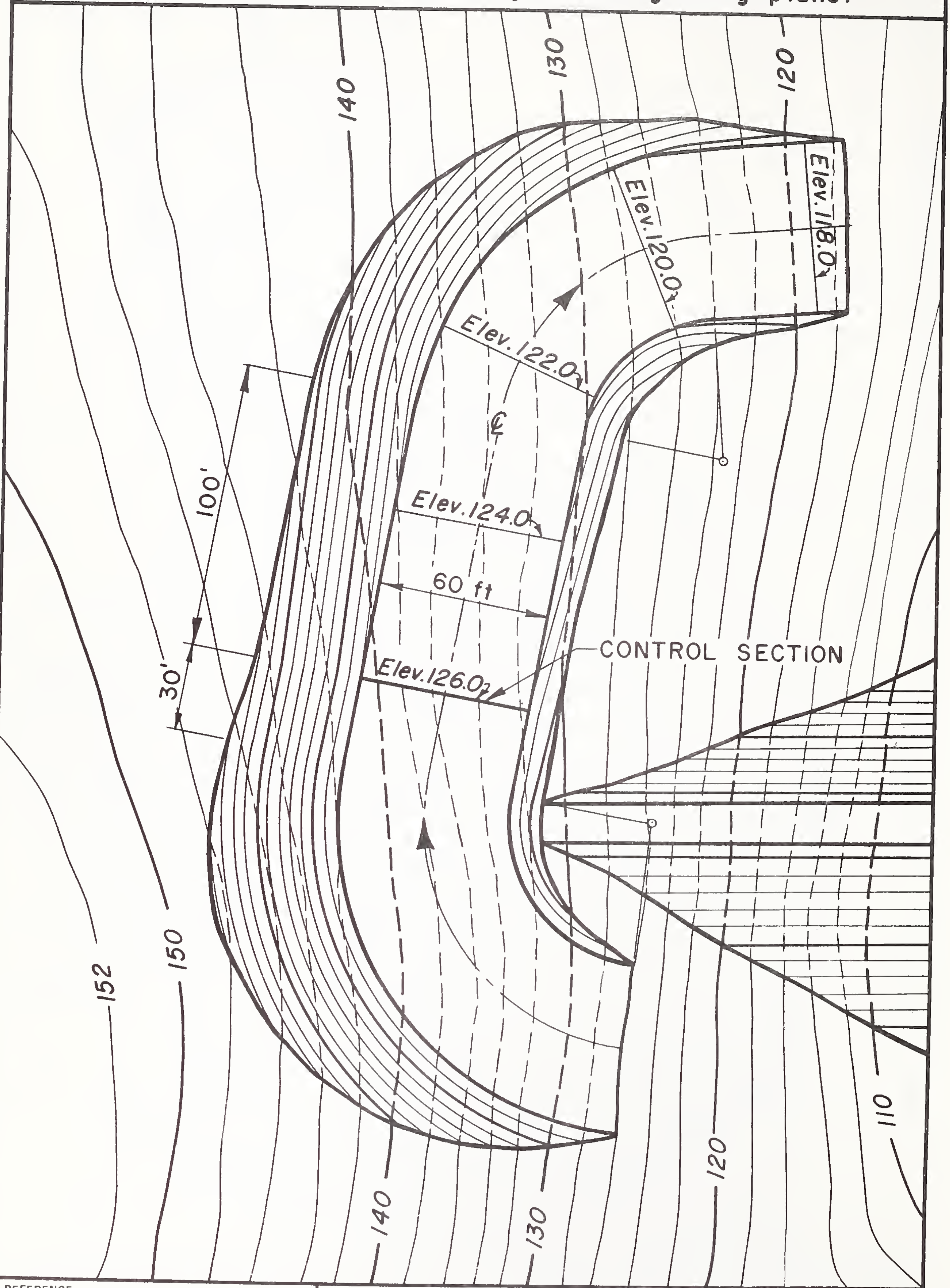
EARTH SPILLWAYS : Typical layout and grading plans.



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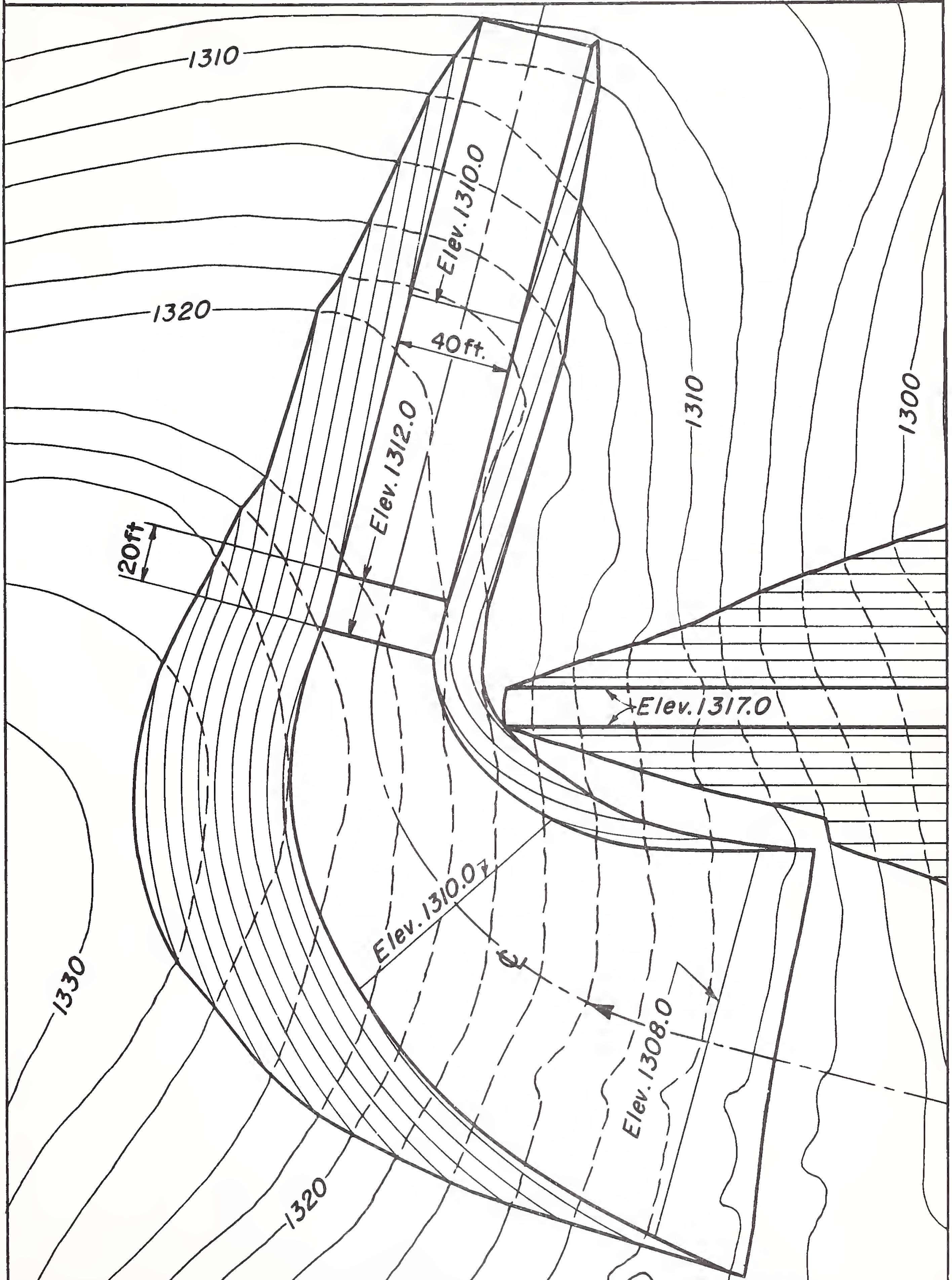
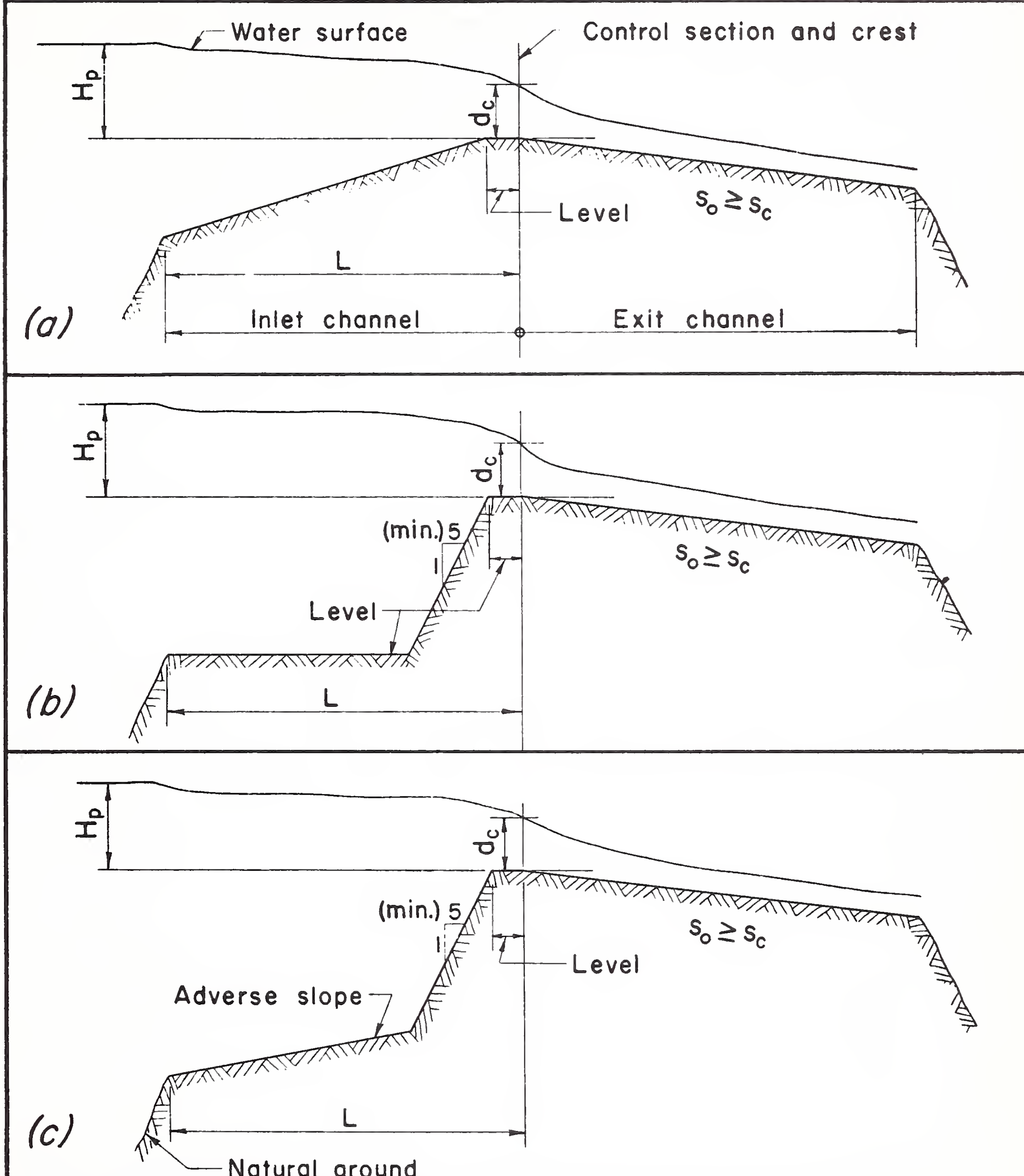
EARTH SPILLWAYS: Typical layout and grading plans.



REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES- 99 SHEET 5 OF 5 DATE 10-1-56

EARTH SPILLWAYS: TYPICAL CONDENSED PROFILES ALONG ℄ OF EARTH SPILLWAY



H_p = Difference in elevation between the crest at the control section and the water surface in the reservoir

d_c = Critical depth

S_o = Bottom slope of the constructed channel

S_c = Critical slope

L = Length of the inlet channel

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES-100 SHEET 1 OF 1 DATE 4-7-55

The following equations apply to a rectangular section. It is assumed that the cross section is a rectangle at the control section and that the same cross section exists throughout the constructed exit channel.

$$Q = wq_c \quad \ldots \quad (1)$$

$$q_c = g^{1/2}\left(\frac{2}{3}\right)^{3/2} H_{ec}^{3/2} = 3.087\ H_{ec}^{3/2} = \frac{v_c^3}{g} \quad \ldots \quad (2)$$

$$H_{ec} = 1.5\ d_c \quad \ldots \quad (3)$$

$$d_c = v_c^2 \div g \quad \ldots \quad (4)$$

$$w = \frac{Qg}{v_c^3} = 32.16\ \frac{Q}{v_c^3} \quad \ldots \quad (5)$$

$$H_{ec} = \frac{3}{2g}\ v_c^2 = 4.66 \times 10^{-2}\ v_c^2 \quad \ldots \quad (6)$$

$$s_c = \frac{g}{1.486^2} \cdot \frac{n^2}{d_c^{1/3}} = 14.56\ \frac{n^2}{d_c^{1/3}} \quad \ldots \quad (7)$$

$$s_c = \frac{g^{4/3}}{1.486^2} \cdot \frac{n^2}{v_c^{2/3}} = 46.32\ \frac{n^2}{v_c^{2/3}} \quad \ldots \quad (8)$$

$$s_c = \frac{g^{10/9}}{1.486^2} \cdot \frac{n^2}{q_c^{2/9}} = 21.42\ \frac{n^2}{q_c^{2/9}} \quad \ldots \quad (9)$$

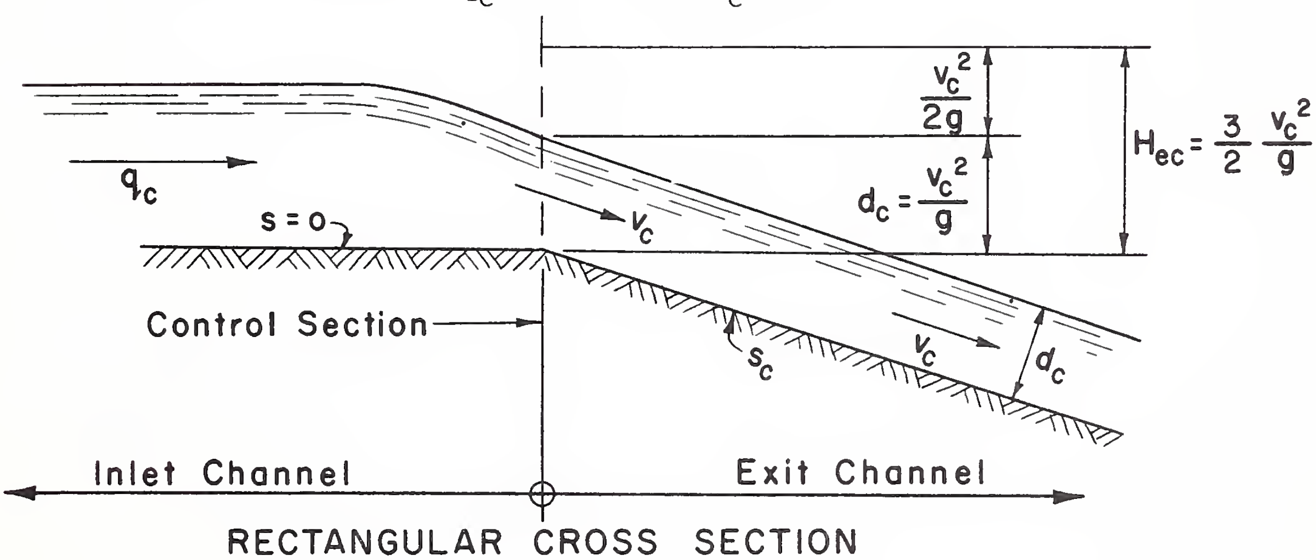


FIGURE 1

For any channel and control section of the same cross section, there exists a different and unique critical depth d_c and critical slope s_c for each different discharge q. Let us assign subscripts to denote different discharges.

Then

$$\frac{s_{c1}}{s_{c2}} = \left(\frac{n_1}{n_2}\right)^2 \left(\frac{q_2}{q_1}\right)^{2/9} \quad \ldots\ldots\ldots\ldots \quad (10)$$

$$\frac{s_{c1}}{s_{c2}} = \left(\frac{n_1}{n_2}\right)^2 \left(\frac{v_{c2}}{v_{c1}}\right)^{2/3} \quad \ldots\ldots\ldots\ldots \quad (11)$$

$$\frac{v_{c1}}{v_{c2}} = \left(\frac{q_1}{q_2}\right)^{1/3} \quad \ldots\ldots\ldots\ldots \quad (12)$$

In the above equations s_{c_1} and v_{c_1} are associated with q_1, etc.

If the slope below the control section is steeper than the critical slope for a particular discharge, the flow will be accelerated, the depth will decrease, and a uniform flow condition will be approached rapidly. In most earth spillways, uniform flow conditions will be approximated within 50 feet of the control section. Assume that uniform flow conditions are reached and that the depth and velocity for this condition are respectively d_e and v_e as shown in figure 2.

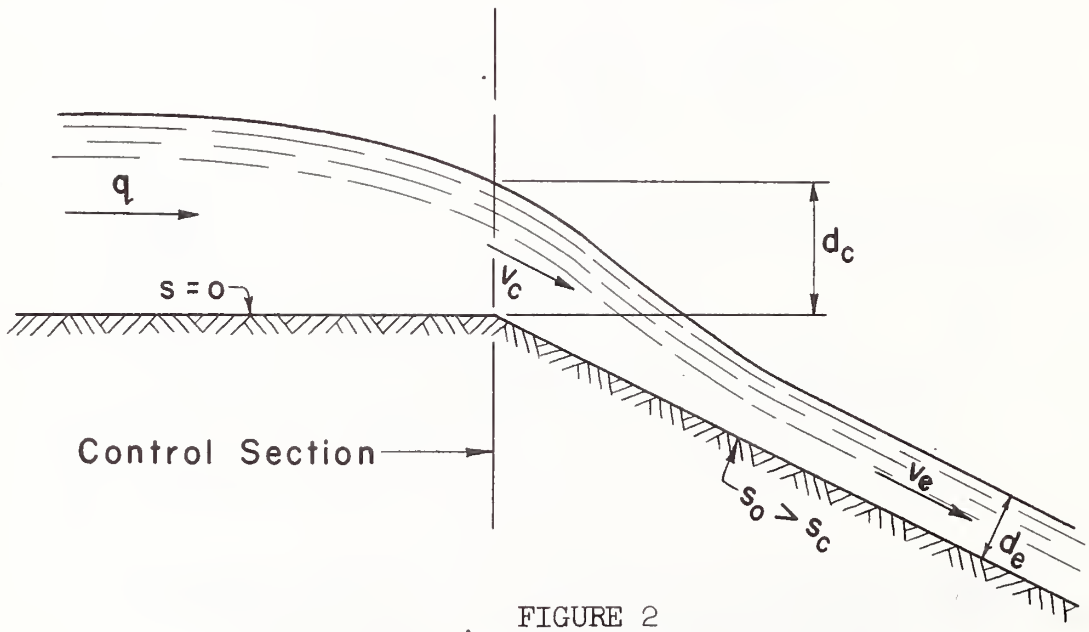


FIGURE 2

Then

$$\frac{s_o}{s_c} = \left(\frac{v_e}{v_c}\right)^{10/3} \quad \ldots\ldots\ldots\ldots \quad (13)$$

Thus to find the maximum allowable slope for a particular discharge and for the maximum allowable velocity in the exit channel v_{em}, the procedure by steps is

1. For the given discharge compute s_c from equation (9) or read it directly from drawing ES-98, sheet 3 of 4.

2. Compute s_o from equation (13) with v_{em} substituted for v_e.

Call the difference in elevation between the control section and the water surface in the reservoir H_p (in feet) and the length of the inlet channel L (in feet).

An approximate value of H_p can be found from the following equation.

$$H_p = H_{ec}(1 + \alpha L) \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (14)$$

where

$$\alpha = \frac{4.315\ n^2}{H_e{}^{4/3}} = \frac{257\ n^2}{v_c{}^{8/3}} \quad \ldots\ldots\ldots\ldots\ldots \quad (15)$$

Where spillway storage may be ignored the design becomes relatively simple.

An approximate solution for this case can be made on the basis of the formulas given above and the following assumptions:

(a) The control section is rectangular and without side friction.

(b) The exit channel has straight alignment and grade and the same cross section as the control section.

(c) A conservative estimate of friction head loss in the inlet channel is acceptable.

Use of the approximate design equations is illustrated in the following example No. 1.

Let b = the bottom width of the trapezoidal section which has been approximated hydraulically by an equivalent rectangular section having a bottom width w.

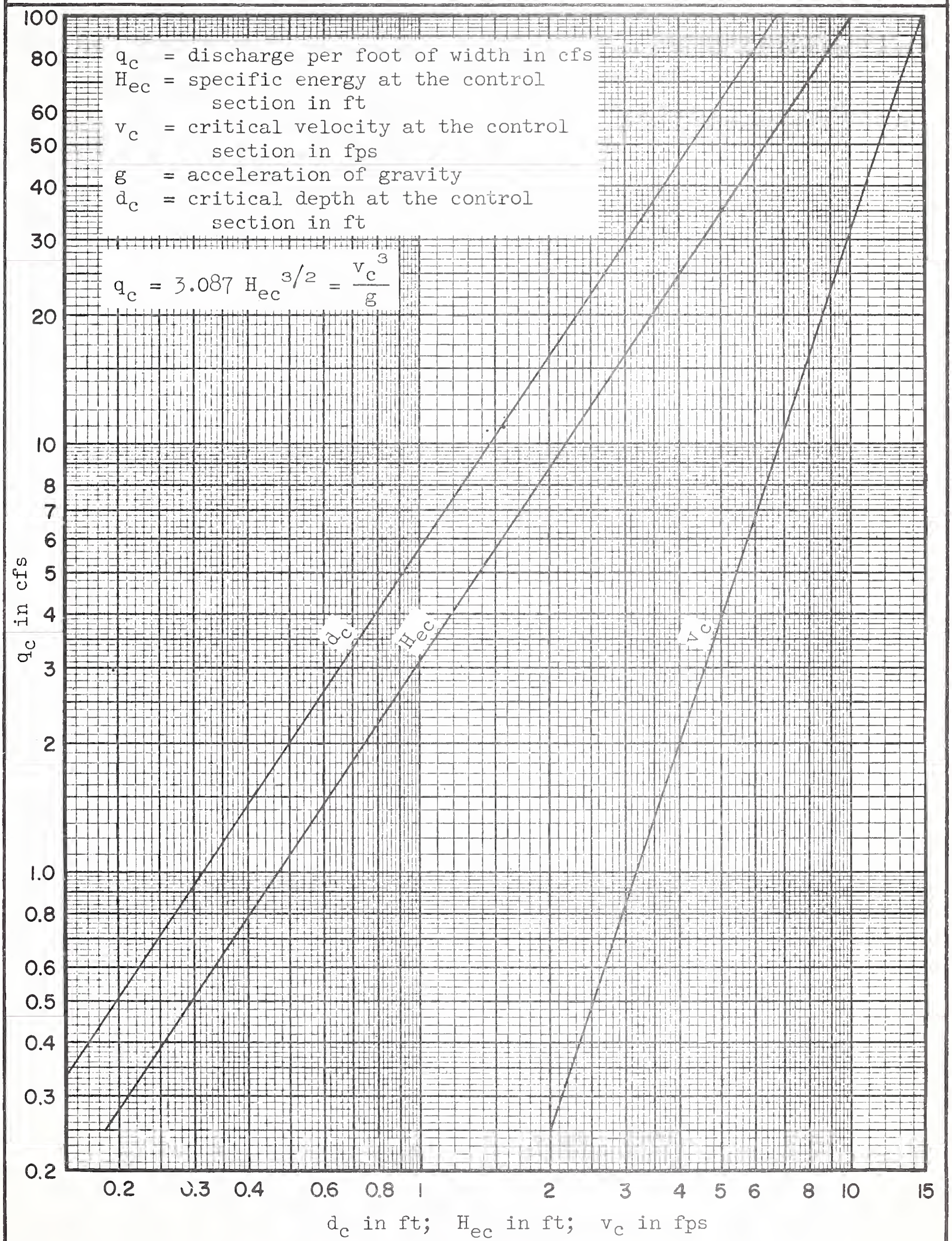
Then

$$b = w - zH_{ec} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (16)$$

where Q = the total discharge, cfs
w = the width of the equivalent rectangular section, ft
q_c = the critical discharge per foot of width, cfs
H_{ec} = the specific energy at the control section, ft

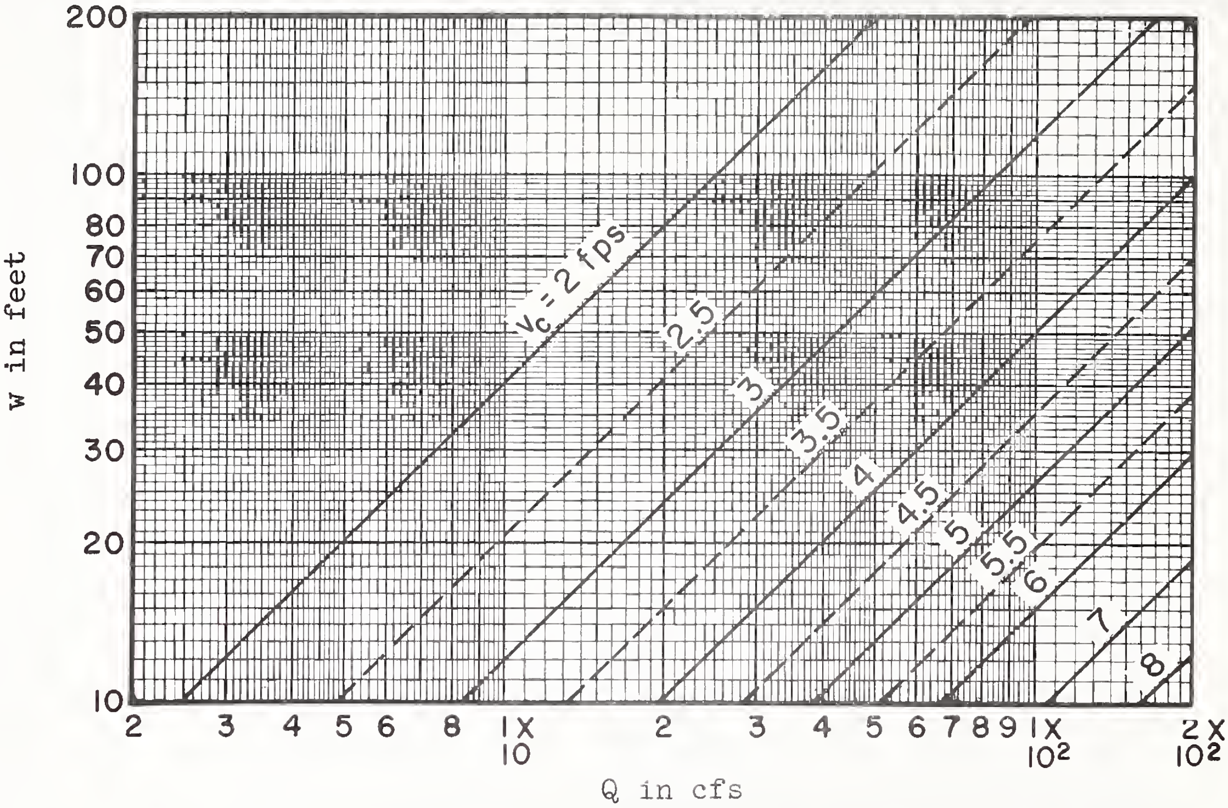
d_c = critical depth at the control section, ft
v_p = permissible velocity, fps
v_e = velocity in exit channel based on the assumption of uniform flow, fps
v_{em} = maximum allowable velocity in the exit channel, fps
v_c = critical velocity at the control section, fps
k_c = ratio $v_c \div v_p$
k_e = ratio $v_e \div v_p$
k_{em} = ratio $v_{em} \div v_p$
g = acceleration of gravity = 32.16 ft/sec^2
s_c = critical slope; i.e., the slope which will just sustain the critical discharge q_c at critical depth d_c, ft/ft
L = length of inlet channel along center line, ft
n = Manning's roughness coefficient
z = the side slope ratio (horizontal divided by vertical)

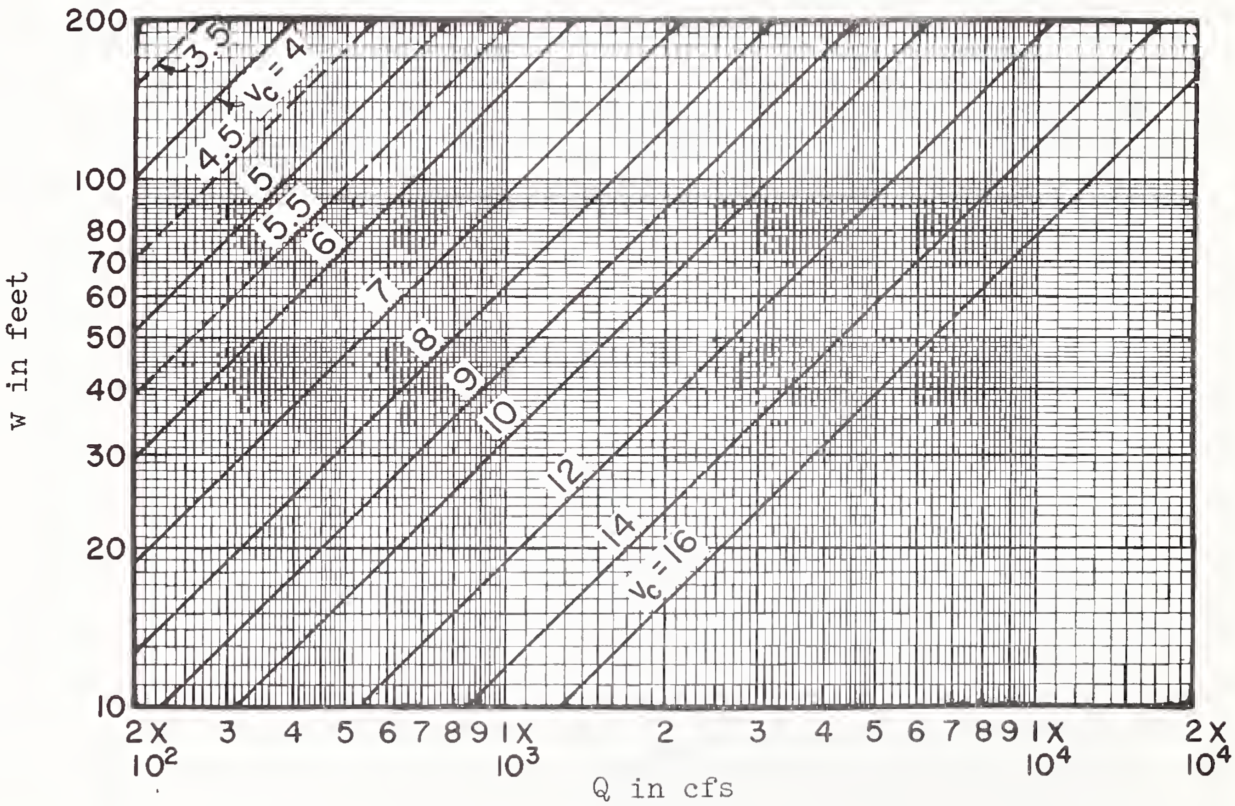
EARTH SPILLWAYS : Plot of q_c vs d_c; q_c vs H_{ec}; q_c vs v_c



REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES-98 SHEET 1 OF 4 DATE 4-27-55

EARTH SPILLWAYS : PLOT OF RELATION BETWEEN w AND Q FOR VARIOUS VALUES OF v_c





$$w = 32.16 \frac{Q}{v_c^{3}}$$

w = width of equivalent rectangular section in ft
Q = total discharge in cfs
v_c = critical velocity at the control section in fps

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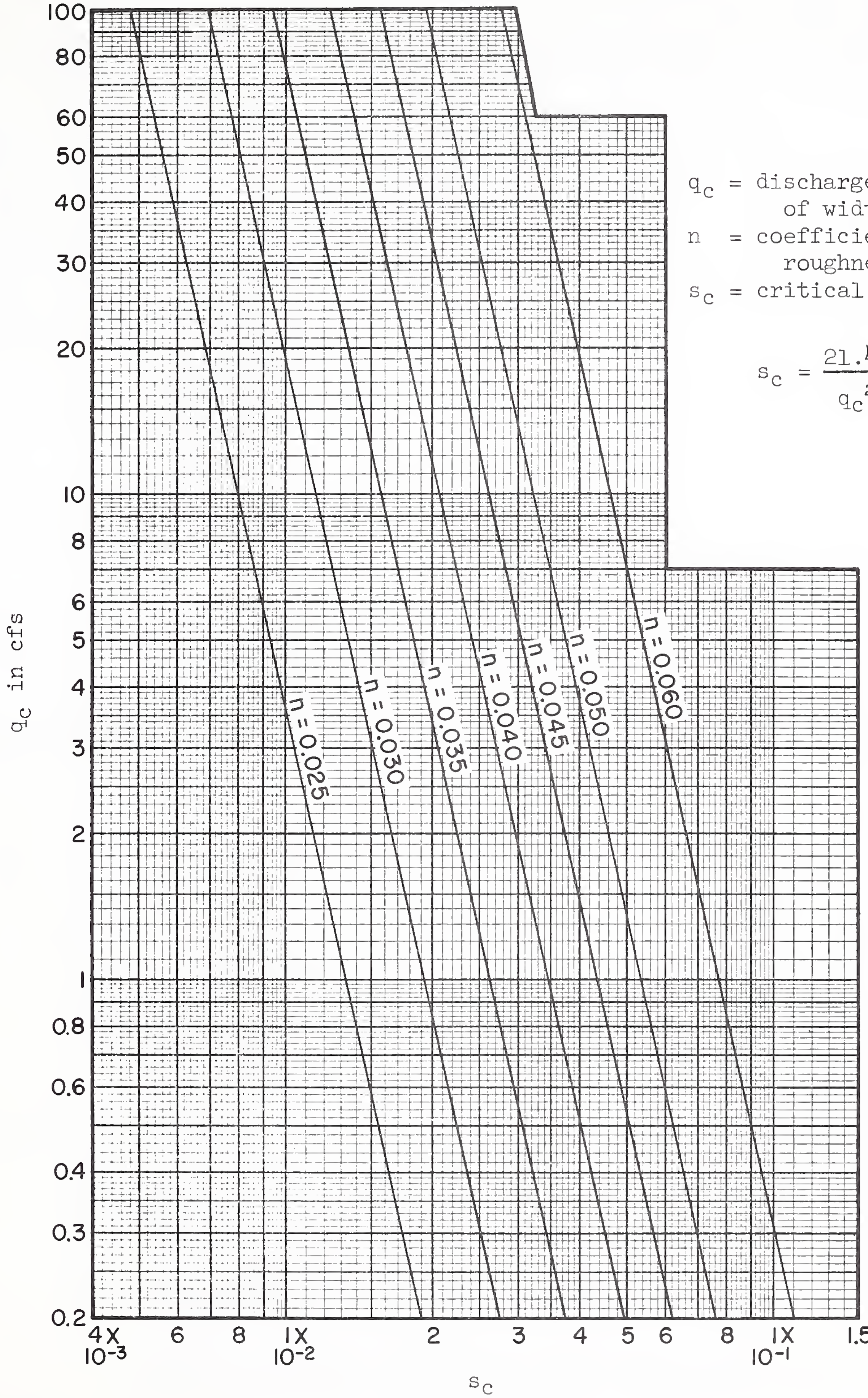
EARTH SPILLWAYS : PLOT OF s_c VS. q_c



q_c = discharge per foot of width in cfs

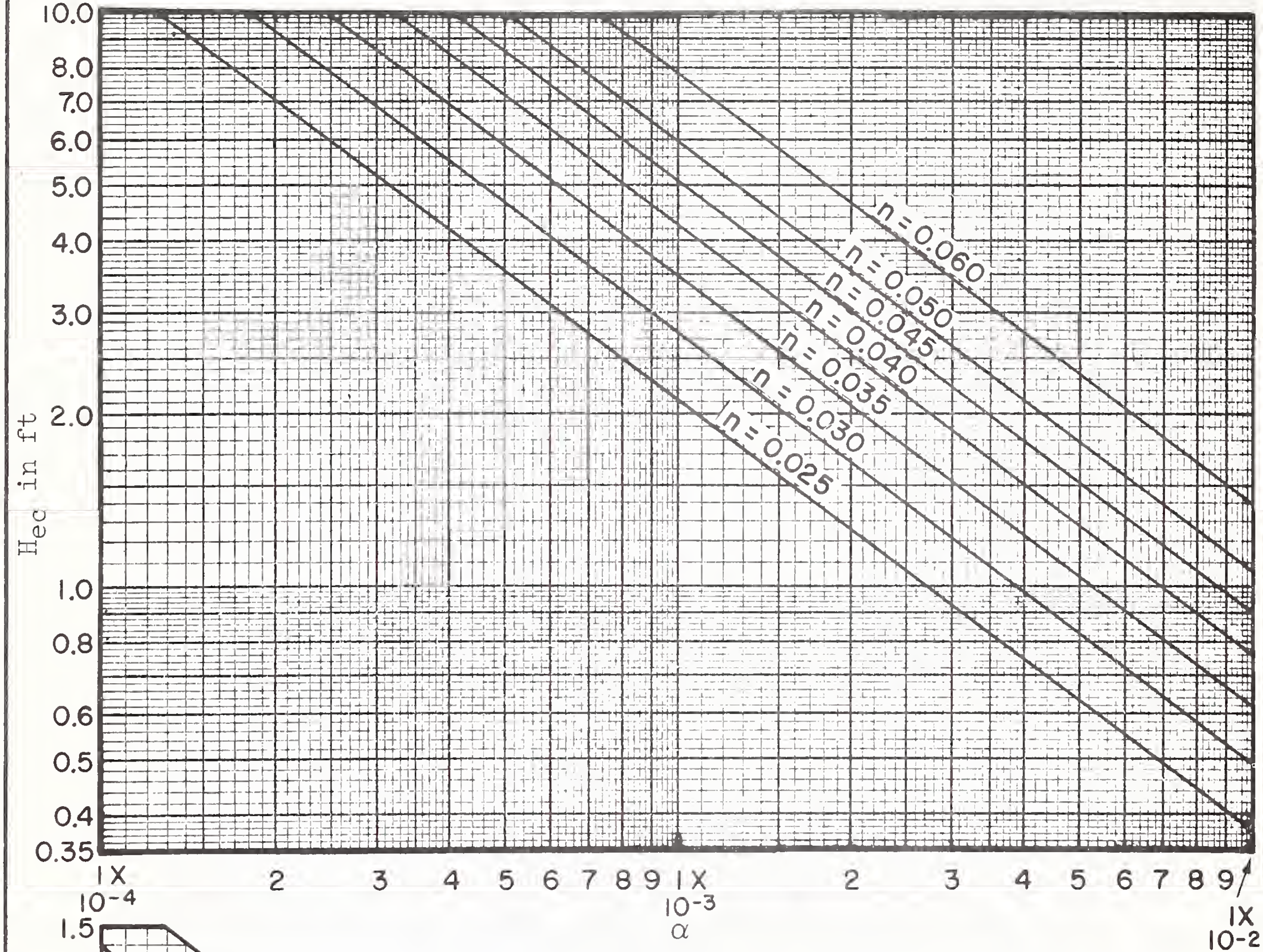
n = coefficient of roughness

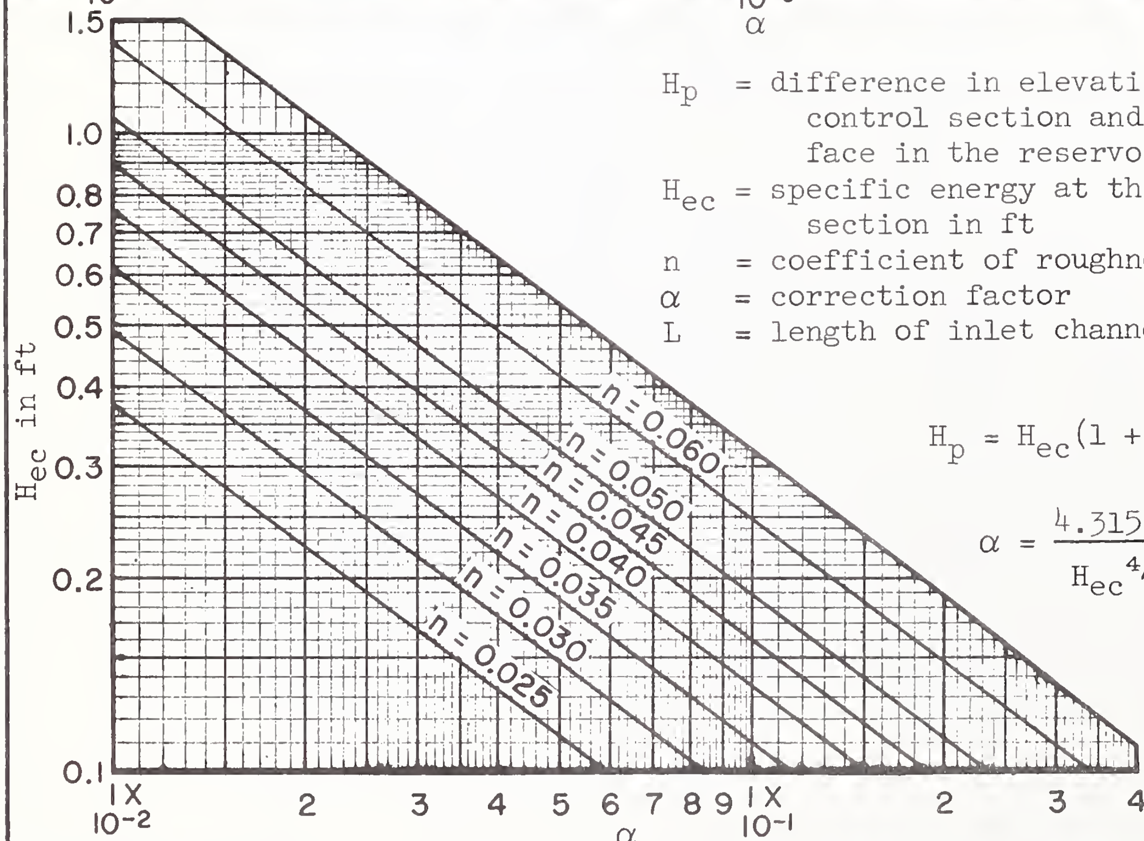
s_c = critical slope

$$s_c = \frac{21.42\ n^2}{q_c^{2/9}}$$

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES- 98 SHEET 3 OF 4 DATE 4-28-55

EARTH SPILLWAYS: PLOT OF RELATION BETWEEN α AND H_{ec} FOR GIVEN VALUES OF n





H_p = difference in elevation between control section and water surface in the reservoir in ft

H_{ec} = specific energy at the control section in ft

n = coefficient of roughness

α = correction factor

L = length of inlet channel in ft

$$H_p = H_{ec}(1 + \alpha L)$$

$$\alpha = \frac{4.315\ n^2}{H_{ec}^{4/3}}$$

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES-98 SHEET 4 OF 4 DATE 4-8-55

EARTH SPILLWAYS: TEN-THIRDS POWERS OF NUMBERS

No.	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	.0000	.0000	.0000	.0000	.0000	.0000	.0001	.0001	.0002	.0003
0.1	.0005	.0006	.0009	.0011	.0014	.0018	.0022	.0027	.0033	.0039
0.2	.0047	.0055	.0064	.0074	.0086	.0098	.0112	.0127	.0144	.0162
0.3	.0181	.0202	.0224	.0248	.0274	.0302	.0332	.0364	.0398	.0433
0.4	.0472	.0512	.0555	.0600	.0648	.0699	.0751	.0807	.0866	.0928
0.5	.0992	.1060	.1131	.1205	.1282	.1363	.1448	.1536	.1627	.1722
0.6	.1822	.1925	.2032	.2144	.2259	.2379	.2503	.2632	.2765	.2903
0.7	.3046	.3193	.3345	.3502	.3665	.3833	.4006	.4185	.4368	.4558
0.8	.4753	.4953	.5161	.5373	.5592	.5817	.6048	.6287	.6529	.6782
0.9	.7039	.7302	.7574	.7852	.8136	.8429	.8727	.9035	.9349	.9671
1.0	1.000	1.034	1.069	1.103	1.141	1.177	1.214	1.252	1.293	1.332
1.1	1.374	1.416	1.459	1.503	1.548	1.593	1.641	1.687	1.737	1.785
1.2	1.836	1.888	1.940	1.994	2.048	2.103	2.161	2.217	2.277	2.338
1.3	2.396	2.459	2.522	2.586	2.654	2.719	2.786	2.856	2.928	2.996
1.4	3.070	3.144	3.218	3.294	3.371	3.452	3.531	3.610	3.694	3.779
1.5	3.865	3.948	4.036.	4.129	4.219	4.310	4.402	4.499	4.592	4.692
1.6	4.792	4.893	4.995	5.099	5.203	5.308	5.415	5.527	5.636	5.750
1.7	5.861	5.978	6.096	6.215	6.335	6.457	6.584	6.708	6.833	6.946
1.8	7.097	7.225	7.360	7.497	7.634	7.773	7.913	8.054	8.202	8.346
1.9	8.497	8.644	8.797	8.952	9.108	9.267	9.425	9.585	9.747	9.910
2.0	10.08	10.25	10.42	10.60	10.76	10.94	11.12	11.30	11.49	11.67
2.1	11.86	12.05	12.24	12.43	12.63	12.82	13.02	13.23	13.43	13.64
2.2	13.85	14.06	14.27	14.49	14.71	14.92	15.15	15.37	15.60	15.83
2.3	16.06	16.30	16.53	16.77	17.01	17.26	17.50	17.75	18.00	18.25
2.4	18.51	18.77	19.03	19.29	19.55	19.83	20.10	20.37	20.65	20.92
2.5	21.21	21.49	21.78	22.07	22.35	22.66	22.95	23.25	23.55	23.86
2.6	24.17	24.48	24.79	25.11	25.43	25.76	26.08	26.41	26.74	27.07
2.7	27.41	27.75	28.09	28.44	28.78	29.14	29.50	29.84	30.21	30.57
2.8	30.94	31.32	31.69	32.06	32.43	32.82	33.20	33.59	33.99	34.39
2.9	34.77	35.18	35.58	35.99	36.41	36.82	37.23	37.66	38.08	38.51
3.0	38.94	39.38	39.82	40.26	40.70	41.15	41.60	42.06	42.51	42.97
3.1	43.44	43.90	44.38	44.85	45.33	45.82	46.31	46.80	47.29	47.79
3.2	48.29	48.79	49.29	49.82	50.32	50.85	51.37	51.90	52.43	52.97
3.3	53.51	54.05	54.60	55.15	55.70	56.25	56.82	57.38	57.96	58.52
3.4	59.11	59.68	60.26	60.86	61.45	62.05	62.65	63.25	63.87	64.48
3.5	65.09	65.72	66.34	66.98	67.62	68.24	68.89	69.54	70.19	70.85
3.6	71.50	72.17	72.85	73.51	74.18	74.87	75.55	76.25	76.95	77.63
3.7	78.34	79.05	79.76	80.48	81.20	81.92	82.66	83.39	84.13	84.88
3.8	85.62	86.38	87.14	87.91	88.66	89.43	90.21	90.99	91.78	92.58
3.9	93.37	94.17	94.98	95.79	96.61	97.42	98.25	99.08	99.92	100.8
4.0	101.6	102.4	103.2	104.2	105.1	105.9	106.7	107.7	108.6	109.4
4.1	110.3	111.3	112.1	113.0	113.8	114.9	115.8	116.6	117.7	118.6
4.2	119.5	120.6	121.4	122.3	123.4	124.3	125.2	126.3	127.2	128.4
4.3	129.3	130.2	131.3	132.3	133.4	134.3	135.5	136.4	137.6	138.5
4.4	139.5	140.7	141.6	142.8	143.8	145.0	145.9	147.1	148.1	149.3
4.5	150.6	151.5	152.8	153.8	155.0	156.0	157.3	158.5	159.5	160.8
4.6	161.8	163.1	164.4	165.4	166.7	167.7	169.0	170.3	171.3	172.7
4.7	174.0	175.0	176.4	177.7	179.0	180.1	181.4	182.8	183.9	185.2
4.8	186.6	188.0	189.1	190.4	191.8	193.2	194.3	195.7	197.1	198.5
4.9	199.9	201.1	202.5	203.9	205.3	206.8	208.2	209.4	210.8	212.3

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES-101 SHEET 1 OF 2 DATE 5-5-55

EARTH SPILLWAYS: TEN-THIRDS POWERS OF NUMBERS

No.	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
5.0	213.7	215.2	216.7	218.2	219.6	220.8	222.3	223.8	225.3	226.8
5.1	228.3	229.8	231.3	232.9	234.4	235.9	237.5	239.0	240.6	242.1
5.2	243.7	245.2	246.8	248.4	250.0	251.5	253.1	254.7	256.3	257.9
5.3	259.5	261.1	262.8	264.4	266.0	268.0	269.6	271.3	272.9	274.6
5.4	276.2	277.9	279.6	281.6	283.2	284.9	286.6	288.3	290.0	292.1
5.5	293.8	295.5	297.2	298.9	301.0	302.8	304.5	306.3	308.0	310.1
5.6	311.9	313.6	315.4	317.6	319.3	321.1	323.3	325.1	326.9	329.1
5.7	330.9	332.7	334.5	336.7	338.6	340.4	342.6	344.5	346.7	348.6
5.8	350.4	352.7	354.6	356.5	358.7	360.6	362.9	364.8	367.1	369.0
5.9	370.9	373.3	375.2	377.5	379.5	381.8	383.8	386.1	388.1	390.5
6.0	392.4	394.8	396.8	399.2	401.2	403.6	405.6	408.0	410.1	412.5
6.1	414.5	417.0	419.4	421.5	423.9	426.0	428.5	431.0	433.1	435.6
6.2	437.6	440.2	442.7	444.8	447.3	449.9	452.0	454.5	457.1	459.2
6.3	461.8	464.4	466.6	469.2	471.8	473.9	476.5	479.2	481.8	484.0
6.4	486.6	489.3	492.0	494.2	496.8	499.5	502.2	504.9	507.2	509.9
6.5	512.6	515.3	518.0	520.3	523.0	525.8	528.5	531.3	534.1	536.4
6.6	539.2	542.0	544.8	547.6	550.4	553.2	556.0	558.4	561.2	564.1
6.7	566.9	569.8	572.6	575.5	578.4	581.3	584.2	587.1	590.0	592.9
6.8	595.8	598.8	601.7	604.7	607.6	610.6	613.6	616.5	619.5	622.5
6.9	625.5	628.5	631.5	634.5	637.6	640.6	643.6	646.7	649.7	652.8
7.0	656.4	659.5	662.5	665.6	668.7	671.8	675.0	678.1	681.2	684.9
7.1	688.0	691.2	694.3	697.5	700.7	704.4	707.6	710.8	714.0	717.2
7.2	720.9	724.1	727.4	730.6	734.4	737.7	740.9	744.2	748.0	751.3
7.3	754.6	757.9	761.8	765.1	768.4	772.3	775.6	779.0	782.3	786.2
7.4	789.6	793.0	796.9	800.3	803.7	807.7	811.1	815.1	818.5	822.0
7.5	826.0	829.4	832.9	836.9	840.4	844.5	848.0	852.1	855.6	859.1
7.6	863.2	866.7	870.8	874.4	878.5	882.1	886.3	889.8	894.0	897.6
7.7	901.8	905.4	909.6	913.2	917.5	921.1	925.4	929.0	933.3	937.0
7.8	941.3	944.9	949.3	953.0	957.3	961.6	965.3	969.7	973.4	977.8
7.9	982.2	986.0	990.4	994.1	998.6	1003	1007	1011	1016	1020
8.0	1024	1028	1032	1037	1041	1045	1050	1054	1059	1063
8.1	1067	1072	1076	1080	1085	1090	1094	1098	1103	1108
8.2	1112	1116	1121	1126	1130	1134	1139	1144	1149	1153
8.3	1157	1162	1167	1172	1176	1181	1186	1190	1195	1200
8.4	1205	1210	1215	1219	1224	1229	1234	1238	1243	1248
8.5	1253	1258	1263	1268	1273	1278	1283	1288	1293	1298
8.6	1303	1308	1313	1318	1324	1329	1334	1339	1344	1349
8.7	1354	1359	1364	1370	1375	1380	1386	1391	1396	1402
8.8	1407	1412	1418	1423	1428	1434	1439	1445	1450	1455
8.9	1461	1466	1472	1478	1483	1488	1494	1499	1505	1511
9.0	1516	1522	1528	1534	1539	1544	1550	1556	1562	1567
9.1	1573	1579	1585	1591	1597	1602	1608	1614	1620	1626
9.2	1631	1637	1643	1649	1656	1661	1667	1674	1679	1685
9.3	1692	1698	1704	1710	1716	1722	1728	1735	1741	1746
9.4	1753	1759	1766	1772	1778	1784	1790	1797	1803	1810
9.5	1816	1822	1828	1835	1841	1848	1854	1861	1867	1874
9.6	1880	1887	1893	1900	1906	1913	1920	1926	1933	1940
9.7	1947	1953	1960	1967	1973	1980	1987	1994	2001	2007
9.8	2014	2021	2028	2035	2041	2048	2055	2062	2069	2077
9.9	2083	2090	2098	2105	2111	2119	2126	2133	2140	2147
10.0	2155	2162	2169	2176	2184	2190	2198	2205	2213	2219

REFERENCE	U. S. DEPARTMENT OF AGRICULTURE SOIL CONSERVATION SERVICE ENGINEERING DIVISION - DESIGN SECTION	STANDARD DWG. NO. ES-101 SHEET 2 OF 2 DATE 5-5-55

EXAMPLE NO. 1

Given: $Q = 120$ cfs; $v_p = 5.5$ fps; $k_c = 1$; $k_{em} = 1.2$; $z = 3$; $n = 0.04$; $L = 60$ ft. Inlet channel is on zero grade.

Find:
1. The required width of the spillway.
2. The minimum slope below the control section.
3. The maximum slope below the control section.
4. The difference in elevation between the crest of the earth spillway and the water surface in the reservoir H_p.

Procedure by steps using equations:

1. Compute $v_c = k_c v_p = (1)(5.5) = 5.5$ fps.

2. Compute $v_{em} = k_{em} v_p = (1.2)(5.5) = 6.6$ fps.

3. Compute w from equation (5).

   $$w = (32.16)(Q \div v_c{}^3) = (32.16)(120 \div 5.5^3) = 23.2 \text{ ft}$$

4. Compute H_{ec} from equation (6).

   $$H_{ec} = 0.0466\, v_c{}^2 = (0.0466)(5.5)^2 = 1.41 \text{ ft}$$

5. Compute b from equation (16).

   $$b = w - zH_{ec} = 23.2 - (3)(1.41) = \underline{19.0 \text{ ft (ans)}}$$

6. Compute $s_{o_{min.}} = s_c$ from equation (8).

   $$s_c = (46.32)(n^2 \div v_c{}^{2/3}) = (46.32)(0.04^2 \div 5.5^{2/3}) = \underline{2.38 \times 10^{-2} \text{ (ans)}}$$

7. Compute $s_{o_{max.}}$ from equation (13).

   $$s_{o_{max.}} = s_c(v_{em} \div v_c)^{10/3} = (2.38 \times 10^{-2})(1.2)^{10/3} = \underline{4.36 \times 10^{-2} \text{ (ans)}}$$

8. Compute α from equation (15).

   $$\alpha = \frac{257\, n^2}{v_c{}^{8/3}} = \frac{(257)(0.04)^2}{5.5^{8/3}} = 0.004363$$

9. Compute H_p from equation (14).

   $$H_p = H_{ec}(1 + \alpha L) = (1.41)\left[1 + (0.004363)(60)\right] = (1.41)(1 + 0.262)$$
   $$= \underline{1.78 \text{ ft (ans)}}$$

Procedure by steps using drawing ES-98:

1. Compute $v_c = k_c v_p = (1)(5.5) = 5.5$ fps.

2. Compute $v_{em} = k_{em} v_p = (1.2)(5.5) = 6.6$ fps.

3. For $v_c = 5.5$, read from ES-98, sheet 1, $q_c = 5.2$ cfs and $H_{ec} = 1.41$ ft.

4. For $v_c = 5.5$ and $Q = 120$, from ES-98, sheet 2, read $w = 23$ ft.

5. Compute b from equation (16).

   $b = w - zH_{ec} = 23 - (3)(1.41) = 18.7$ ft (say 19 ft)

6. For $q_c = 5.2$ (step 3) and $n = 0.04$ from ES-98, sheet 3, read $s_c = 2.38 \times 10^{-2} = s_{o_{min.}}$

7. Compute $s_{o_{max.}}$ from equation (13) substituting $s_{o_{max.}}$ for s_o and v_{em} for v_e. Read the ten-thirds power of $(v_{em} \div v_c)$ from ES-101.

   $s_{o_{max.}} = s_c(v_{em} \div v_c)^{10/3} = (2.38 \times 10^{-2})(1.2)^{10/3} = 4.36 \times 10^{-2}$

8. Read α from sheet 4 of ES-98; for $H_{ec} = 1.41$ and $n = 0.04$, $\alpha = 4.35 \times 10^{-3}$

9. Compute H_p from equation (14).

   $H_p = H_{ec}(1 + \alpha L) = 1.41\left[1 + (4.35 \times 10^{-3})(60)\right] = \underline{1.78 \text{ ft (ans)}}$

It should be noted that this approximate solution is direct because of the approximations assumed in the development of the design equations.

A closer approximation of H_p can be made by the following procedure for any discharge for which the control section actually operates as a control section. The procedure by steps is

1. Compute the specific energy H_{ec} at the control section.

2. Divide the inlet into reaches of fixed length. Ends of reaches should be placed at each break in grade in the inlet channel. The accuracy of the final result depends upon the length of the reaches selected; the shorter the reaches the more accurate the result. Reasonable accuracy should be attained in most cases if the first 50 to 60 feet of the inlet channel is divided into reaches 10 to 20 feet long. Reach lengths may be increased to 40 to 80 feet upstream from the first 50 to 60 feet of inlet channel and if the cross-sectional area of flow has been significantly increased by increasing the depth or width of the inlet channel as indicated in drawing ES-99, sheets 1, 2, and 3 of 4, the length of the reaches above the point of increase may be made 80 to 120 feet long. These suggested reach lengths are obviously arbitrary and may be shortened or lengthened depending upon the desired accuracy. Seldom will it be advisable to make the first two reaches longer than 30 feet each.

3. Compute the approximate value of H_p. This is done in a step by step procedure; starting from the specific energy at the control section, compute a hypothetical elevation at the upstream end of the first reach from the equation

$$E_1 = E_2 + \frac{q^2 n^2}{2.208\ E_2^{10/3}} (\ell_2 - \ell_1) - s_o(\ell_2 - \ell_1) \quad . . (17)$$

where E_1 = hypothetical elevation at upstream end of reach
E_2 = hypothetical elevation at downstream end of reach
q = $Q \div w$ = discharge per foot of equivalent width, cfs
Q = discharge, cfs
w = $b + zE_2$ = equivalent bottom width, ft
b = bottom width of trapezoidal section, ft
z = side slope ratio (horizontal ÷ vertical)
n = Manning's friction factor
ℓ_2 = station of lower end of reach (stationing in the direction of flow)
ℓ_1 = station of upper end of reach
s_o = slope of bottom of inlet channel through reach. (Adverse slopes are negative.)

The value of E_2 for the first reach will be the specific energy at the control section H_{ec}.

E_1 for the first reach becomes E_2 for the second, and the computation is repeated reach by reach progressing upstream. These computations should be tabulated as in example 2 which follows.

4. The required value of H_p is found by adding the summation of $s_o(\ell_2 - \ell_1)$ values through the inlet channel to the value of E_1 for the most upstream reach.

EXAMPLE NO. 2

Given: Q = 120 cfs; b = 19 ft; z = 3; n = 0.04; L = 60 ft; approach channel is flat (zero grade)

Find: The value of H_p.

Solution by steps:

1. $\frac{Q}{b} = \frac{120}{19} = 6.32$; $\frac{z}{b} = \frac{3}{19} = 0.158$; From ES-24, $d_c = 1.02$;

   $a_c = (19)(1.02) + (3)(1.02)^2 = 22.5\ ft^2$; $v_c = \frac{Q}{a_c} = \frac{120}{22.5} = 5.33$ fps;

   $H_{ec} = d_c + \frac{v_c^2}{2g} = 1.46$ ft

2. Since there are no breaks in grade in the inlet channel, divide the channel into two reaches of 15 ft and one upstream reach of 30 ft.

3. Tabulate the computations as follows.

Column	1		2	3	4	5	6	7
Row	Reach ℓ_2	Reach ℓ_1	$\ell_2 - \ell_1$	E_2	$E_2^{10/3}$	b	$w = b + zE_2$	$q = \frac{Q}{w}$
1	0+60	0+45	15	1.46	3.52	19	23.38	5.133
2	0+45	0+30	15	1.54	4.22	19	23.62	5.080
3	0+30	0+00	30	1.61	4.89	19	23.83	5.036

Column	8	9	10	11
Row	q^2	$\frac{q^2n^2}{2.208} \times \frac{1}{E_2^{10/3}}$	9 $\times (\ell_2 - \ell_1)$	E_1 = 3 + 10 + $s_o(\ell_2 - \ell_1)$
1	26.34	5.42×10^{-3}	0.0813	1.54
2	25.81	4.43×10^{-3}	0.0665	1.61
3	25.36	3.76×10^{-3}	0.1128	1.72

4. Since the inlet channel is flat $s_o(\ell_2 - \ell_1) = 0$ and $H_p = 1.72$ ft.

EXAMPLE NO. 3

Given: Q = 120 cfs; b = 10 ft at control section; z = 3; n = 0.04; L = 60 ft. The slope and the bottom width of the inlet channel are as given in the table below.

Find: The value of H_p.

Solution by steps:

1. From example No. 2, H_e = 1.46 ft.
2. Divide channel into reaches as indicated in the table in step 3.
3. Tabulate computations.

Column	1		2	3	4	5	6	7	8
Row	Reach		$\ell_2 - \ell_1$	s_o	$s_o(\ell_2 - \ell_1)$	$\Sigma s_o(\ell_2 - \ell_1)$	E_2	$E_2^{10/3}$	b
	ℓ_2	ℓ_1							
1	0+60	0+50	10	0	0	0	1.46	3.52	19
2	0+50	0+40	10	0	0	0	1.51	3.95	19
3	0+40	0+30	10	-0.20	-2.00	-2.00	1.56	4.40	19
4	0+30	0+00	30	-0.02	-0.60	-2.60	3.60	71.50	7

Column	9	10	11	12	13	14
Row	$w = b + zE_2$	$q = \frac{Q}{w}$	q^2	$\frac{q^2n^2}{2.208} \times \frac{1}{E_2^{10/3}}$	12 $\times (\ell_2 - \ell_1)$	E_1 = 6 + 13 − 4
1	23.38	5.133	26.34	$5.42\text{x}10^{-3}$	0.0542	1.5142
2	23.53	5.100	26.01	$4.77\text{x}10^{-3}$	0.0477	1.5577
3	23.68	5.068	25.68	$4.24\text{x}10^{-3}$	0.0423	3.6023
4	17.80	6.740	45.43	$4.60\text{x}10^{-4}$	0.0138	4.2138

$$\frac{n^2}{2.208} = 0.725\text{x}10^{-3}$$

4. H_p = 4.21 − 2.60 = 1.61 ft.

More refined methods for the design of earth spillways to carry a specified peak discharge rate without consideration of spillway storage are demonstrated in example No. 4. In this procedure the approximations listed in paragraphs (a), (b), and (c) of part 5.1 are not permitted; i.e., the actual cross section at the control section is used to compute the hydraulic properties of the section and the difference in elevation between the crest of the

earth spillway and the water surface in the reservoir is found by computing the water surface profile upstream from the crest, through the inlet section, to the reservoir. The procedure by steps is as follows:

1. Determine the permissible velocity v_p and select the proper value of k_c and k_e depending upon the physical characteristics of soil and vegetation to be expected and the frequency of operation of the spillway.

2. For a given side slope ratio, find the value of the bottom width at the control section which will convey the peak design discharge rate at a critical depth such that the mean velocity at critical depth v_c is equal to $k_c v_p$. This is accomplished by assuming trial values of bottom width at the control section b and then computing

(a) d_c = critical depth at the control section in ft. This value is most easily obtained from drawing ES-24, National Engineering Handbook, Section 5, Hydraulics, which also contains the equation for critical depth in trapezoidal channels.

(b) a_c = cross-sectional area of flow at the control section in ft^2.

(c) v_c = the critical velocity at the control section in fps.

$$v_c = Q \div a_c \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (18)$$

Trial values of b are chosen until one is found such that v_c, computed as outlined above, is essentially equal to $k_c v_p$ for the values of k_c and v_p chosen to control the design.

3. Find the minimum permissible slope of the constructed exit channel below the control section so that a control section will exist for the peak design discharge. For a control section to exist the normal depth of flow in the exit channel must not exceed the critical depth, hence the minimum permissible slope is the critical slope s_c which will just maintain steady uniform flow at critical depth. Since critical flow is usually very unsteady, in actual design, a minimum slope should be chosen slightly greater than the theoretical minimum in order to provide at least a slight acceleration below the control section. Since the critical slope is directly proportional to the square of Manning's roughness coefficient, good design requires the use of the highest reasonable value of n that can be expected to prevail in the determination of the slope of the exit channel below the control section. In this discussion it is assumed that the exit channel is straight in alignment and grade and of the same cross section as the control section.

The critical slope is readily found by the use of drawing ES-55, National Engineering Handbook, Section 5, Hydraulics. First compute the ratio $d_c \div b$ and for this value and the given side slope ratio z read the value of $(nQ) \div (b^{8/3}s^{1/2})$ from ES-55. Then with n, Q, and b known, the value of s_c can be found.

4. Next compute the maximum permissible slope in the constructed exit channel assuming that uniform flow conditions are attained, that the maximum allowable velocity is equal to $v_{em} = k_{em}v_p$, and that the bottom width and side slope ratio of the control section are maintained throughout the constructed exit channel. With Q, v_{em}, b, and z known, the normal depth d_n can be computed. Next compute $d_n \div b$ and from ES-55 find the value of $(nQ) \div (b^{8/3}s^{1/2})$ from which the value of s can be computed.

5. The final step is to compute the water surface profile from the control section, where the depth is d_c, in an upstream direction to the reservoir. The method of computing such water surface profiles is explained in Supplement A of Section 5 of the National Engineering Handbook on Hydraulics.

Application of the above procedure to a specific problem follows:

EXAMPLE NO. 4

Given: $Q = 120$ cfs; $v_p = 5.5$ fps; $k_c = 1$; $k_{em} = 1.2$; $z = 3$; $n = 0.04$.

Find:
1. The required width of the spillway.
2. The minimum slope below the control section.
3. The maximum slope below the control section through the constructed exit channel.
4. The difference in elevation between the crest of the earth spillway and the water surface in the reservoir.

Procedure by steps:

1. The values of v_p, k_c, and k_{em} are given in this example.
2. $v_c = k_c v_p = (1)(5.5) = 5.5$ fps.

   The cross-sectional area of flow at the control section

   $a_c = Q \div v_c = 120 \div 5.5 = 21.82$ ft^2 (required)

Prepare table as follows:

b	$Q \div b$	$z \div b$	d_c	a_c	
20	6.00	0.150	0.984	22.59	
18	6.67	0.167	1.055	22.32	
16	7.50	0.1875	1.116	21.59	
17	7.06	0.1765	1.079	21.85	okeh

3. $d_c \div b = 1.079 \div 17 = 0.0635$

   From ES-55, read $(nQ) \div (b^{8/3} s_c^{1/2}) = 0.01605$

$$s_c^{1/2} = \frac{(4 \times 10^{-2})(1.2 \times 10^2)}{(1.605 \times 10^{-2})(1.91 \times 10^3)} = 1.565 \times 10^{-1}; \quad s_c = 2.45 \times 10^{-2}$$

4. $v_{em} = k_{em} v_p = (1.2)(5.5) = 6.6$ fps

   $a = Q \div v_{em} = 120 \div 6.6 = 18.18$ ft^2 $= (17 + 3\ d_e)\ d_e$

   $3\ d_e^2 + 17\ d_e = 18.18$; $\quad d_e = 0.92$ ft; $\quad d_e \div b = 0.0541$

   $(nQ) \div (b^{8/3} s_n^{1/2}) = 0.01213$

$$s_e^{1/2} = \frac{(4 \times 10^{-2})(1.2 \times 10^2)}{(1.213 \times 10^{-2})(1.91 \times 10^3)} = 2.071 \times 10^{-1}; \quad s_e = 4.29 \times 10^{-2}$$

5. Step 5 is omitted in this example because the computation of water surface profiles is explained and illustrated in the National Engineering Handbook, Section 5, Hydraulics.

Where the effect of spillway storage is to be included in the design of an earth spillway, it is necessary to know the stage-discharge curve for all of the spillways including the earth spillway so that the flood routing can be accomplished. (See National Engineering Handbook, Section 5, Hydraulics.) Since the available spillway storage above the crest of the earth spillway is a function of the stage, and the discharge through the earth spillway is a function of both stage and width of the earth spillway, even a rough procedure for approximating the minimum allowable width of the earth spillway becomes a valuable tool in the design process.

At least two design inflow hydrographs are needed to proportion the spillways (mechanical and earth) which are planned as integral parts of an earth dam. A particular inflow hydrograph is routed through the lower mechanical spillway and reservoir to fix the elevation of the crest of the earth spillway. A larger hydrograph is then routed through both spillways; to perform this routing it is necessary to know the stage-discharge curve for both the mechanical and the earth spillways and for their combined discharge. The stage-discharge curve for the earth spillway is dependent on the layout of the inlet channel, the roughness coefficient n in the inlet channel, and the width and shape of cross section of the control section. For a given inlet channel there is a unique stage-discharge curve; and when the larger flood is routed through this particular spillway unique values of the reservoir stage (and height of earth embankment), water-surface profile, and mean velocities at various cross sections are attained. If the width of the control section is too small the velocities will exceed permissible limits and the stage in the reservoir and hence the height and volume of earth embankment will be excessive. Thus the permissible velocities at the control section play an important part in the determination of the minimum width of the control section.

For the reasons pointed out previously widths of earth spillways greater than the minimum are desirable. A trial value for the minimum allowable width of earth spillway can be found by the following procedure:

1. Find point F (Fig. 3) at which the earth spillway first starts to operate. To do this, route the inflow hydrograph through the mechanical spillway until the storage is equal to the available storage in the reservoir between the normal pool level (crest of the lowest mechanical spillway) and the elevation of the crest of the earth spillway V_{sp} as determined from the stage-storage curve.

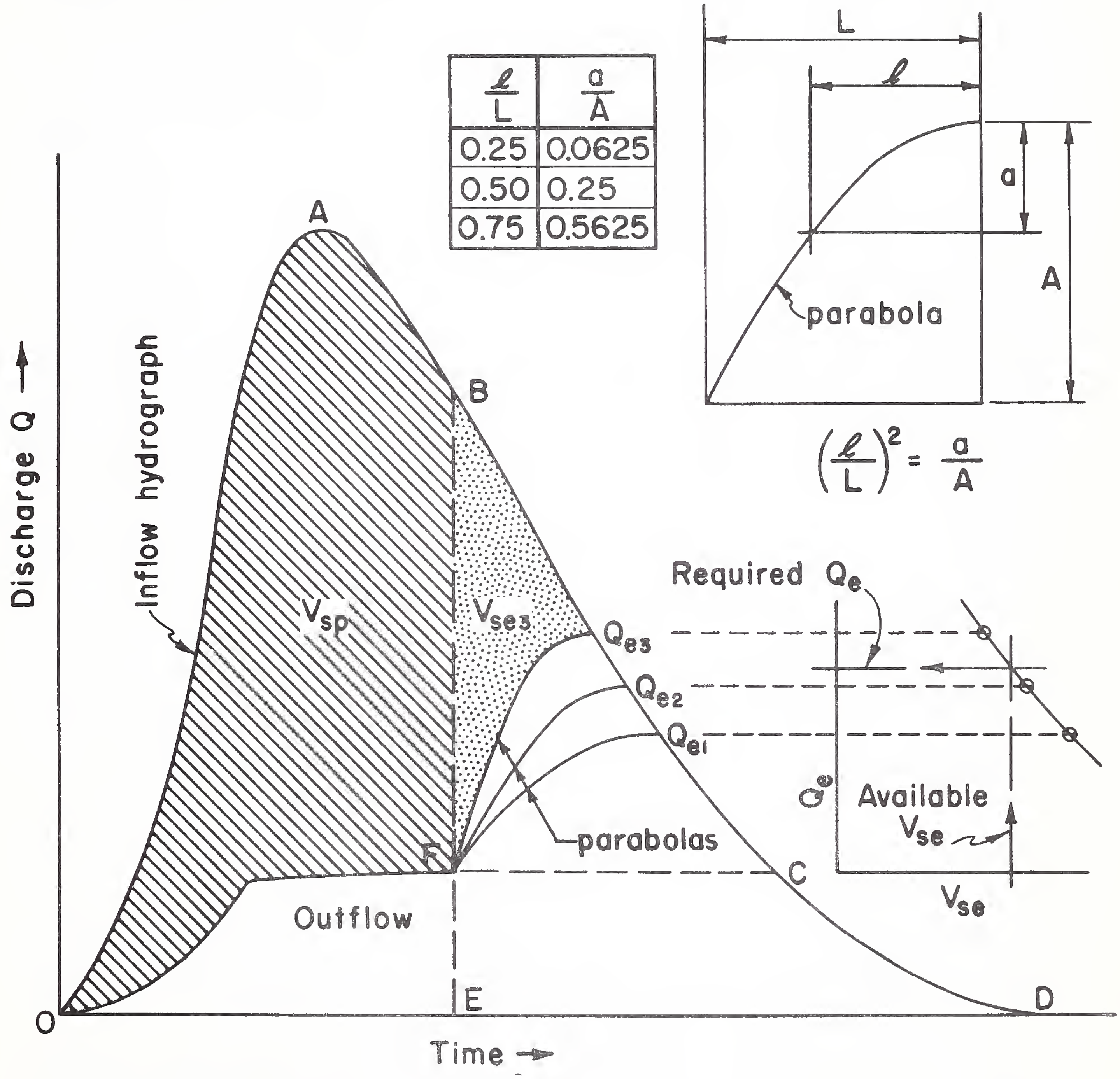


Figure 3

2. Next draw a vertical line EFB and check the area OABFO to see that it is equivalent to V_{sp}.

3. Select the permissible velocity v_p for the earth spillway and $k_c = v_c \div v_p$. Estimate the value of Manning's roughness coefficient n for the maximum depth of flow to be permitted through the inlet channel at permissible velocities. Also measure from the plans the approximate center line length of the inlet channel L.

4. Compute H_p from equations (14) and (15).

5. Find the available earth spillway storage V_{se} from the stage-storage curve for the reservoir for the value of H_p found in step 4.

6. Construct a horizontal line from point F to point C, Figure 3. This line becomes the base line for measuring rates of discharge through the earth spillway.

7. Find a value of the peak rate of discharge through the earth spillway Q_e such that the earth spillway storage V_{se} indicated by the dotted area FBQ_eF is equal to the available storage V_{se} found in step 5. This is done by assuming values of Q_e such as Q_{e1}, Q_{e2}, etc., measuring the associated values of V_{se} and plotting the results to form a curve such as is shown on Fig. 3. From this curve find the value of Q_e corresponding to the available V_{se} as found in step 5.

8. Find the approximate minimum width of the earth spillway from equations (5) and (16) substituting Q_e for Q in equation (5).

After having found the approximate minimum width of the earth spillway, the next design step is to decide on the width of earth spillway to be used in the final design. Having made this decision and having made a layout in accordance with principles previously stated, a more accurate analysis, if necessary, should proceed as illustrated in the following example.

EXAMPLE NO. 5

Given: An earth spillway with b = 120 ft and z = 3 at the control section and throughout the exit channel. The layout of the spillway is as shown in drawing ES-99, sheet 1 of 4. The maximum permissible velocity is v_p = 6 fps, k_c = 1, and k_e must not exceed 1.25; n = 0.04.

Find: The required slope below the control section, the reservoir stage-discharge curve for the spillway, the maximum discharge capacity of the spillway just at the point of overtopping of the earth dam embankment, and the maximum probable velocities associated with this maximum discharge capacity.

Procedure by steps:

1. Assume increasing values of d_c and compute the nine columns of the following table. Columns 1 through 4 can be copied directly from "Hydraulic Tables" for certain values of b or can be computed. The computation of Columns 5 through 8 is self-evident. Column 9 may be computed from equation 5.4-24, page 5.4-10, National Engineering Handbook, Section 5, Hydraulics, or read directly with care from ES-34.

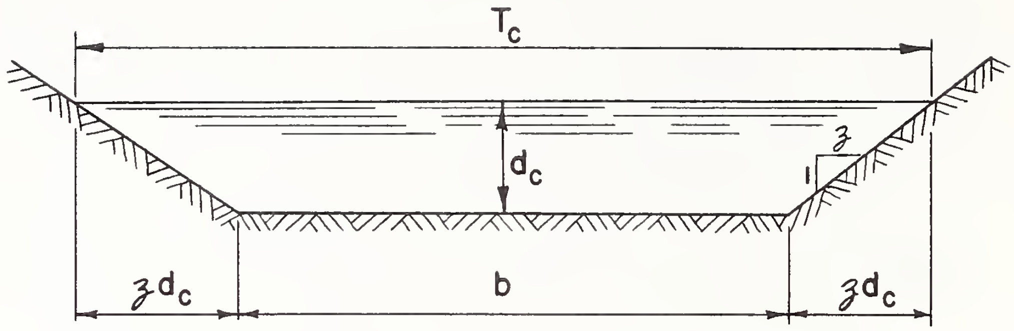
2. Plot critical velocity v_c (Col. 7), critical depth d_c (Col. 1), and critical slope s_c (Col. 9) against discharge Q (Col. 8), as illustrated. Find the value of Q = 850 cfs associated with the value of v_c = 6 fps.

To insure that the control section actually acts as a control section for a major part of the discharge range, it is necessary to arbitrarily select a relatively low discharge rate below which the break in grade at the upstream end of the exit channel ceases to define a control section. In this example this arbitrarily selected discharge was taken as 10 percent of 850 cfs = 85 cfs for which discharge the critical slope is 0.0362 = required slope.

3. With a slope of 0.0362 in the exit channel, acceleration will take place below the control section for all discharges greater than 85 cfs. Assuming that uniform flow conditions are attained, the velocity in the exit channel for a discharge of 850 cfs is found as follows.

$b = 120$; $z = 3$; $n = 0.04$; $\sqrt{g} = \sqrt{32.16} = 5.67$

1	2	3	4	5	6	7	8	9
d_c	a_c	T_c	r	$d_m = \frac{a_c}{T_c}$	$\sqrt{d_m}$	$v_c = \sqrt{g d_m}$	$Q = a_c v_c$	s_c
0.2	24.12	121.2	0.20	0.199	0.446	2.53	61	0.0396
0.4	48.48	122.4	0.40	0.396	0.629	3.56	173	0.0311
0.6	73.08	123.6	0.59	0.592	0.769	4.36	318	0.0278
0.8	97.92	124.8	0.78	0.785	0.886	5.02	491	0.0254
1.2	148.32	127.2	1.16	1.166	1.080	6.12	907	0.0223
1.6	199.68	129.6	1.53	1.541	1.241	7.04	1405	0.0204
2.0	252.00	132.0	1.90	1.909	1.382	7.84	1975	0.0190
2.4	305.28	134.4	2.26	2.272	1.506	8.54	2610	0.0178
2.8	359.52	136.8	2.61	2.629	1.621	9.19	3310	0.0170
3.2	414.72	139.2	2.96	2.979	1.725	9.78	4055	0.0163
3.6	470.88	141.6	3.30	3.327	1.824	10.34	4870	0.0158



CROSS-SECTION AT CONTROL SECTION

$$a_c = (b + zd_c)d_c \;;\quad T_c = b + 2zd_c$$

$$v_c = \frac{Q}{a_c} \qquad d_m = \frac{a_c}{T_c}$$

$$H_{ec} = d_c + \frac{d_m}{2} = d_c + \frac{v_c^2}{2g}$$

FIGURE 4

$$ar^{2/3} = \frac{Qn}{1.486\ s^{1/2}} = \frac{(850)(0.04)}{(1.486)(0.0362)^{1/2}} = 120.2$$

Trial	d	a	r	$r^{2/3}$	$ar^{2/3}$
1	0.8	97.92	0.78	0.847	82.9
2	1.0	123.0	0.97	0.980	120.5

Interpolate between trials 1 and 2.

1.0 -- 120.5
120.2
0.8 -- 82.9

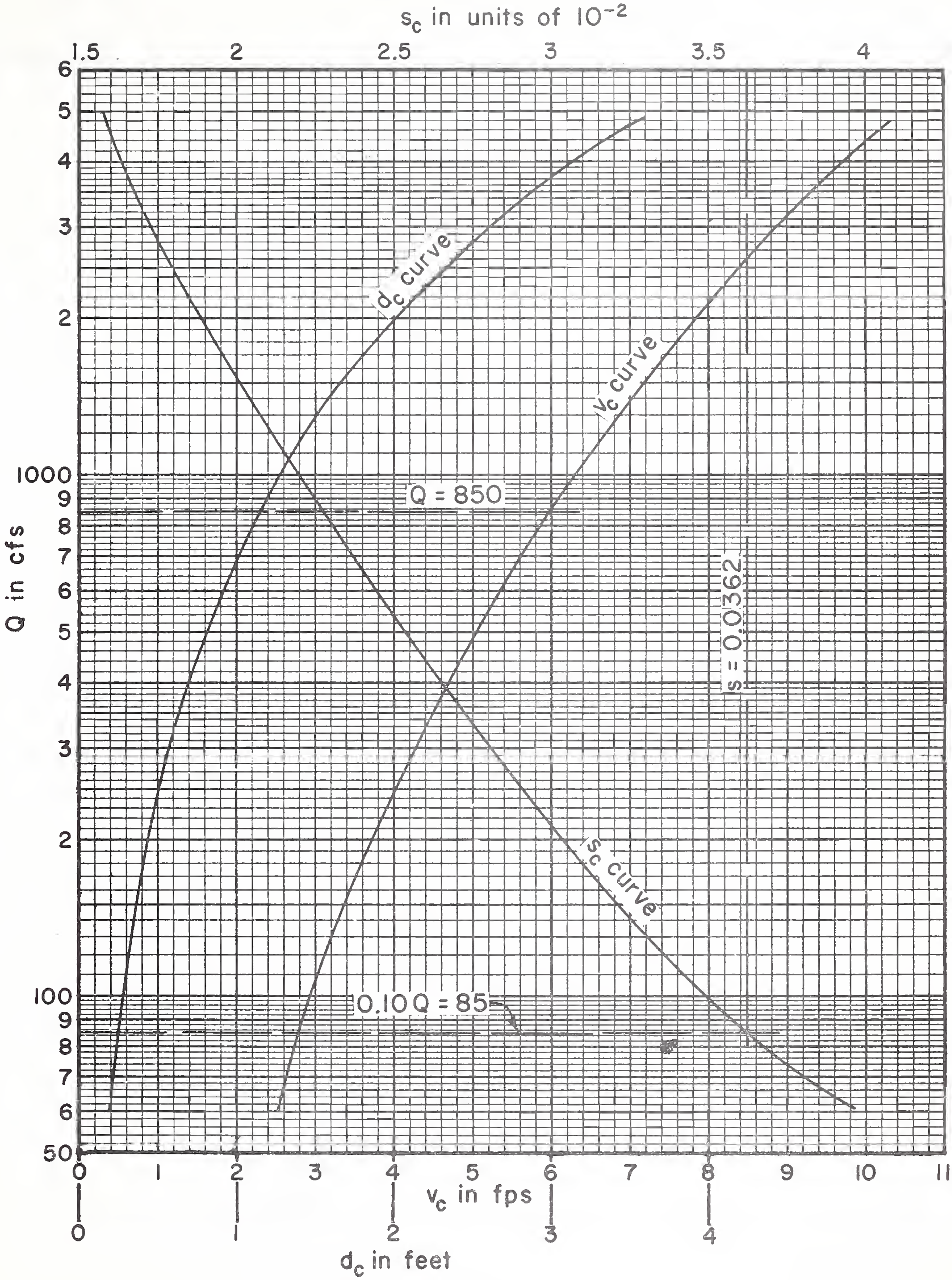
$$\frac{37.3}{37.6} \times 0.2 = 0.1985$$

$$\frac{\quad 0.80}{0.9985} = d \quad \text{(Say 1.0)}$$

$$a = (b + zd)d = 120 + (3)(1.0)\ 1 = 123\ \text{ft}^2$$

$$v_e = 850 \div 123 = 6.91\ \text{fps}$$

$$k_e = \frac{v_e}{v_c} = \frac{6.91}{6} = 1.15 < 1.25\ \text{(okeh)}$$



s_c in units of 10^{-2}
1.5
2
2.5
3
3.5
4
Q in cfs
6
5
4
3
2
1000
9
8
7
6
5
4
3
2
100
9
8
7
6
50
d_c curve
V_c curve
s_c curve
Q = 850
s = 0.0362
0.10 Q = 85
0
1
2
3
4
5
6
7
8
9
10
11
v_c in fps
0
1
2
3
4
d_c in feet

Figure 5

4. Water surface profile computations give the following data for the plotting of the stage-discharge curve:

Discharge Q in cfs	Water-surface Elevation in Reservoir	H_p in ft
85	66.60	0.60
250	67.03	1.03
500	67.49	1.49
750	67.86	1.86
1000	68.18	2.18
1500	68.75	2.75
2000	69.26	3.26
3000	70.18	4.18
4000	71.00	5.00
5300	72.00	6.00

5. The maximum velocity in the exit channel associated with the discharge just at the point of overtopping the earth embankment is computed in a manner similar to step 3 above. With s = 0.0362, Q = 5300 cfs, and n = 0.03, compute the value or $ar^{2/3}$

$$ar^{2/3} = \frac{Qn}{1.486\ s^{1/2}} = \frac{(5300)(0.03)}{(1.486)(0.0362)^{1/2}} = 562$$

Trial	d	a	r	$r^{2/3}$	$ar^{2/3}$	
1	2.4	305.28	2.26	1.722	525	
2	2.6	332.28	2.44	1.812	602	
Check	2.496	319	2.35	1.767	563	(okeh)

2.6 -- 602
562
2.4 -- 525

$$\frac{37}{77} \times 0.2 = 0.096$$

$$\begin{array}{r} 0.096 \\ 2.40 \\ \hline 2.496 = d \end{array}$$

$$v_e = \frac{5300}{319} = 16.6 \text{ fps}$$

A very good approximation of H_p can be made by the following procedure for any discharge for which the control section actually operates as a control section. The procedure is more accurate than the method employing the use of the equivalent rectangular section as expressed by equation (17) and should be used when accuracy is required.

The procedure by steps is:

1. Compute the specific energy at the control section H_{ec}. This is $H_{ec} = d_c + \frac{v_c^2}{2g}$.

2. The inlet channel is divided into reaches in the same manner as previously recommended. Once the inlet reach lengths have been fixed, they should remain the same for each different discharge computation. The entire center-line length of the inlet channel must be considered; that is, the distance from the beginning of the constructed inlet channel to the control section.

3. Compute H_p. This is done in a step by step procedure; starting from the specific energy H_{ec} at the control section, compute the value of E_1 at the upstream end of the first reach from the equation

$$E_1 = E_2 + s(\ell_2 - \ell_1) - s_o(\ell_2 - \ell_1) \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (19)$$

where
E_1 = hypothetical elevation at upstream end of reach
E_2 = hypothetical elevation at downstream end of reach
s = slope
s_o = slope of bottom of inlet channel through the reach (adverse slopes are negative)
ℓ_2 = station at lower end of reach (stationing in direction of flow)
ℓ_1 = station at upper end of reach

The value of E_2 for the first reach will be the specific energy at the control section H_{ec}. E_1 for the first reach becomes E_2 for the second reach and the computation is repeated reach by reach in an upstream direction.

H_p is found by adding the summation of the $s_o(\ell_2 - \ell_1)$ values through the inlet channel to the value of E_1 for the most upstream reach. These computations should be tabulated as in the examples that follow.

4. H_p should be computed for a minimum of three Q values as listed below:

(a) A value of Q close to the minimum value of Q for which the control section of the spillway actually operates as control section. The method of determining this value of Q has been previously explained.

(b) The design Q of the spillway.

(c) A value of Q that is close to the maximum capacity of the spillway, or the value of Q that would have a corresponding value of H_p equal to the elevation of the top of the dam minus the elevation of the spillway crest. A value of Q for this purpose may be obtained by using equations (14), (15), and (16).

It may be desirable to compute H_p for more values of Q in the given range.

5. Plot the associated values of Q and H_p found in step 4 on log-log paper, and connect these points with a curve which will be found to be smooth and nearly straight. This curve may be extrapolated to give the maximum value of Q for the maximum value of H_p and the minimum value of H_p for the minimum value of Q for which the control section actually operates as a control section.

The Cartesian coordinate plot of the stage-discharge curve of the spillway may be made from points on the log-log plot curve of H_p vs Q for the inlet channel of the spillway when the control section is actually operating as a control section.

EXAMPLE NO. 6

Given: b = 120 ft; z = 3; n = 0.04; L = 180 ft; station 1+20 is the beginning of the constructed inlet channel; station 3+00 is the crest of the spillway. The inlet channel grade is flat (zero slope); crest elevation = 66.00; elevation of top of dam = 72.00.

Find: H_p for any given value of Q for which the control section of the spillway actually operates as a control section.

Procedure:

1. The method of establishing the design Q of the spillway and the minimum value of Q for which the control section actually operates as a control section has been previously explained. For this example the minimum value of Q will be 10 percent of the design Q or 85 cfs. The maximum value of Q, the value of Q that would raise the pool elevation to the point of overtopping the dam section six feet above the crest, may be estimated with equations (14), (15), and (16).

2. For the purpose of this example the three Q values selected will be 200 cfs; 850 cfs, and 3000 cfs. The computation of H_p for each of the selected Q values should be tabulated in the same form as the computation of H_p for Q = 3000 cfs which follows.

Tabulation form for $E_1 = E_2 + s(\ell_2 - \ell_1) - s_o(\ell_2 - \ell_1)$:

(a) Column 1 lists the stations of the ends of the reaches considered.

(b) Column 2 lists the reach lengths in feet.

(c) Column 3 lists the elevation E_2 at the downstream end of the reach in feet. The value of E_2 for the first reach will be the specific energy at the control section H_{ec}.

(d) Column 4 lists the bottom width of the downstream trapezoidal section in feet.

(e) Column 5 lists $a_2 = E_2(b + 3\ E_2)$.

(f) Column 6 lists $p_2 = b_2 + 2\ E_2\sqrt{z^2 + 1}$.

(g) Column 7 lists the cross-section factor F_2; assuming that a_2 is the cross-sectional area of the water section of depth E_2, and p_2 is the wetted perimeter of the section of E_2. This is read from ES-76.

(h) Column 8 lists the values of column 7 divided by Manning's coefficient of roughness.

(i) Column 9 is the square of the reciprocal of column 8. This can be read from the double scale of ES-77.

(j) Column 10 is the product of column 9 and Q^2.

(k) Column 11 is the product of column 2 and column 10.

(l) Column 12 lists column 3 plus column 11.

(m) Column 13 lists the channel slope for the reach of column 1.

(n) Column 14 is the product of column 2 and column 13.

(o) Column 15 lists the hypothetical elevation at the upstream end of the reach. This is column 12 minus column 14. The value of E_1 for the first reach becomes E_2 for the second reach and the computation is repeated reach by reach upstream.

(p) Column 16 is the required value of H_p and is found by adding the summation of $s_o(\ell_2 - \ell_1)$ values through the inlet channel to the value of E_1 for the most upstream reach.

E_2 at 3+00 $= d_c + \frac{v^2}{2g} = 2.63 + 1.237 = 3.867$ $\qquad d_c$ (read from ES-24 = 2.63)

Q = 3000 cfs $\qquad \frac{v^2}{2g} = 1.237$

Column	1		2	3	4	5	6	7	8
Row	Reach		$\ell_2 - \ell_1$	E_2	b_2	a_2	p_2	$F_2 = \frac{nQ}{s^{1/2}}$	$\frac{Q}{s^{1/2}} = \frac{F_2}{n}$
	ℓ_2	ℓ_1							
1	3+00	2+70	30	3.867	120	508.8	144.4	1748	4.37×10^4
2	2+70	2+40	30	4.009	120	529.0	145.3	1860	4.65×10^4
3	2+40	1+20	120	4.134	120	547.2	146.1	1960	4.90×10^4

Column	9	10	11	12	13	14	15	16
Row	$\frac{s}{Q^2}$	s	$s(\ell_2 - \ell_1)$	$s(\ell_2 - \ell_1) + E_2$	s_o	$s_o(\ell_2 - \ell_1)$	E_1	H_p
1	5.25×10^{-10}	47.2×10^{-4}	0.142	4.009	0	0	4.009	
2	4.63×10^{-10}	41.6×10^{-4}	0.125	4.134	0	0	4.134	
3	4.16×10^{-10}	37.4×10^{-4}	0.450	4.584	0	0	4.584	
4						Σ	+4.584	4.584

3. In a similar manner, H_p will be 1.24 ft for a Q of 200 cfs and H_p will be 2.43 ft for a Q of 850 cfs.

4. By using log-log paper and plotting

H_p	Q
1.24	200
2.43	850
4.58	3000

The resulting curve may be extrapolated to obtain a Q value of 4900 cfs for the maximum H_p of 6 ft, the point of overtopping the dam; and an H_p value of 0.84 for the minimum Q of 85 cfs for which the control section actually operates as a control section. This log-log plot is shown at the end of example No. 2.

5. The Cartesian coordinate plot of the stage-discharge curve of the spillway may be made from points on the log-log curve of H_p vs Q for the inlet channel of the spillway when the control section is actually operating as a control section.

EXAMPLE NO. 7

Given: $z = 3.0$; $n = 0.04$; $L = 180$ ft. Other channel characteristics are as illustrated in drawing ES-99, sheet 1 of 4.

Find: H_p for any given value of Q for which the control section actually operates as a control section.

Procedure: 1. The method of determining the required Q values has been described in Procedure, Step 1, Example No. 1.

2. The three selected Q values for this example will be 200 cfs, 850 cfs, and 3000 cfs. The computation of H_p for each of the selected Q values should be tabulated in the same form as the computation of H_p for Q = 3000 cfs which follows.

Column	1		2	3	4	5	6	7	8
Row	Reach		$\ell_2 - \ell_1$	E_2	b_2	a_2	p_2	$F_2 = \frac{nQ}{s^{1/2}}$	$\frac{Q}{s^{1/2}} = \frac{F_2}{n}$
	ℓ_2	ℓ_1							
1	3+00	2+70	30	3.867	120	508.8	144.4	1748	$4.37\text{x}10^4$
2	2+70	2+40	30	4.009	120	529.0	145.3	1860	$4.65\text{x}10^4$
3	2+40	1+20	120	10.134	84	1159.4	148.1	6800	$17.00\text{x}10^4$

Column	9	10	11	12	13	14	15	16
Row	$\frac{s}{Q^2}$	s	$s(\ell_2 - \ell_1)$	$s(\ell_2 - \ell_1) + E_2$	s_o	$s_o(\ell_2 - \ell_1)$	E_1	H_p
1	$5.25\text{x}10^{-10}$	$47.2\text{x}10^{-4}$	0.142	4.009	0	0		
2	$4.63\text{x}10^{-10}$	$41.6\text{x}10^{-4}$	0.125	4.134	-0.2	-6.0	10.134	
3	$3.46\text{x}10^{-11}$	$31.14\text{x}10^{-4}$	0.037	10.171	0	0	10.171	
4						-6.0	10.171	4.171

3. In the same computation procedure as above, H_p would be 0.99 for a Q of 200 cfs and H_p would be 2.02 for a Q of 850 cfs.

4. Use log-log paper and plot

H_p	Q
0.99	200
2.02	850
4.17	3000

As illustrated in the following graph, Figure 6, the curve may be extrapolated to obtain a Q value of 5200 cfs for the maximum H_p of 6 ft, the point of overtopping the dam; and an H_p value of 0.70 for the minimum design Q of 85 cfs.

5. The Cartesian coordinate plot of the stage-discharge curve of the spillway may now be made.

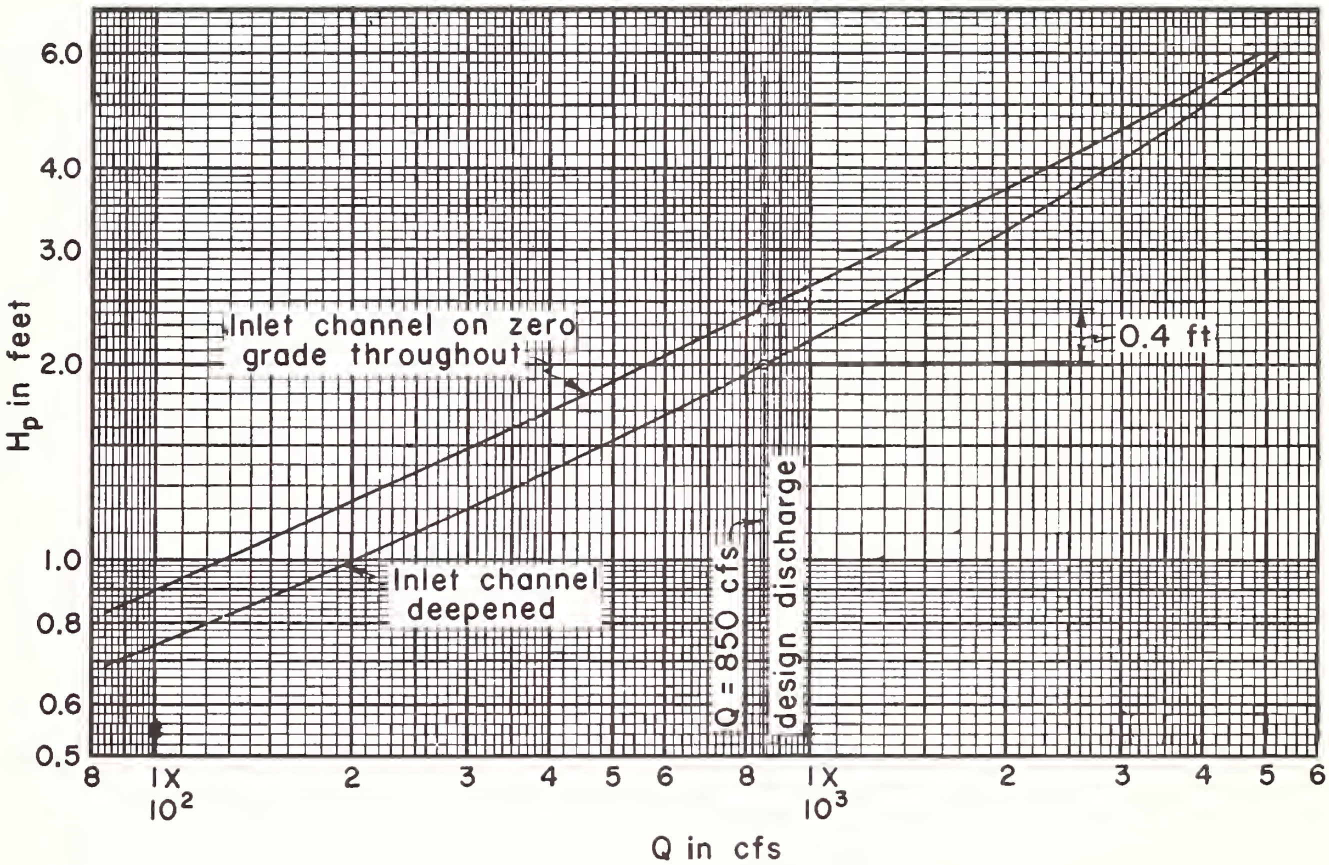


Figure 6

Maintenance. The earth spillway must be maintained in such a way that its full capacity to carry water is available at all times. The wetted perimeter of the channel for a maximum possible discharge must be kept free of brush, debris, and any other possible obstruction. Annual mowing is advisable.

Regular inspection is the first step in maintenance.

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